Distil

What should we watch tonight?

Why sometimes your favourite streaming service just cannot manage to recommend anything interesting

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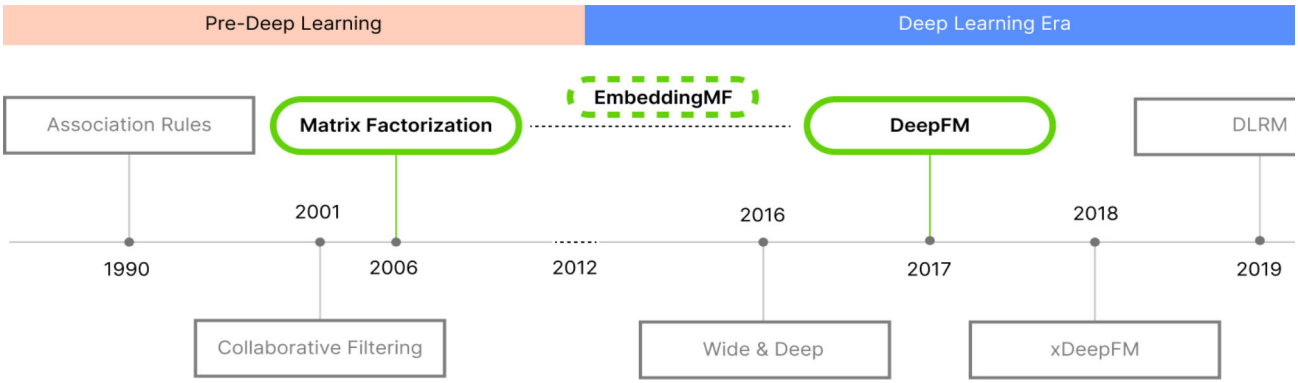


Timeline of recommender systems techniques. The green line highlight the different techniques that the article will describe. Specifically, the green solid line marks the two techniques available in the literature. The dashed green line, identify a custom model built by the authors that connects the two models.

If you have ever used an e-commerce service or a streaming platform, you have already come across something like: "*recommended for you*", or "*other users have also bought this*". Our educational article below will give you an introduction to Recommender Systems (RS), and illustrate how this field currently leverages deep-learning techniques. Our article is meant to foster in the reader a broad inuition of RS, highlight common scenarios causing failure in various RS techniques, and provide a visual understanding of how recommender systems work.

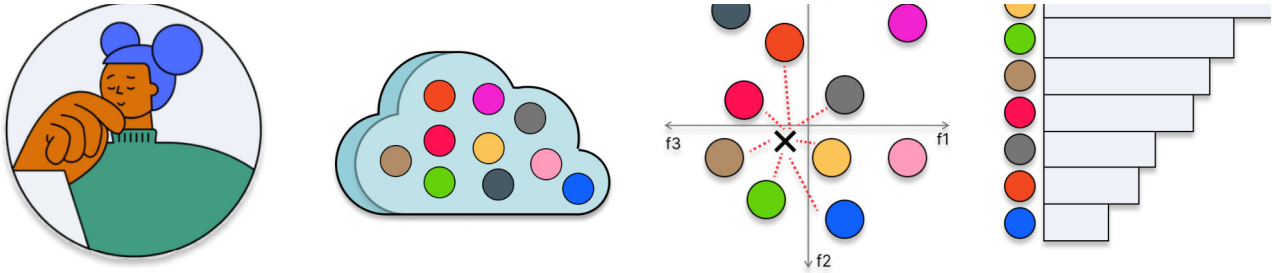
First, we illustrate how Matrix Factorization (MF) works-- a (relatively) simply designed recommender system. Then, we gradually increase the sophistication of our approach, exploring the effect this has on the prediction accuracy of a model. We explain differences in two models' competence in predicting a user's rating of movies they have seen, through various visualizations. Our two models use the MovieLens 1M dataset, selecting different sets of features (columns) per model [18].

Recommender systems: what are they?

Recommender systems are defined as techniques that suggest items for a given user to interact with [1]. Usually, and especially on the web, recommendations are personalized to the specific user interacting with the system. By "*item*", we refer to *what* the system is recommending to the user. This item could be the next movie you watch, song you listen to, the fastest path to your next destination, or your next date!

The recommendation process can be divided into four steps: querying, retrieving, filtering, scoring, and ranking.

This box is interactive! *Hover over the different steps below to learn more*



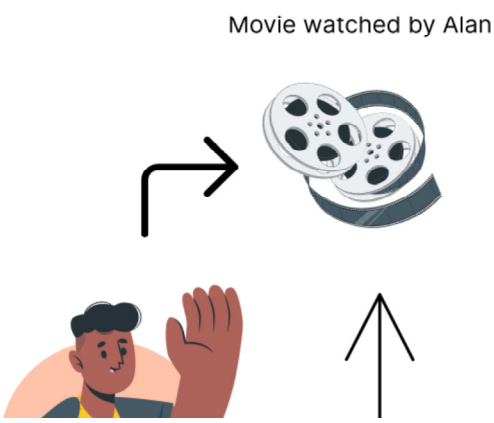
In order to generate recommendations, RS may accept input data like users' preferences, items' characteristics, explicit feedback, implicit feedback... Essentially, any measurable information about the items or users the RS is interested in modeling. When computing recommendations, this information is retrieved from a database and then stored in matrices. The matrix data object optimizes a lot of the computation required to reach a recommendation. Why this happens is beyond the scope of this discussion, but you can read about this process in detail here [10].

The most accessible data are explicit ratings, which include explicit input from the user regarding their level of interest in a product, i.e. the rating a user gives to an item. Usually, explicit feedback can be represented by a *"sparse"* matrix. A sparse matrix is characterized as a matrix where only a small number of fields have non-zero (or non-null) values. Since users are unlikely to rate more than a small number of items from the entirety of those available in a dataset, querying explicit information results in sparse matrices.

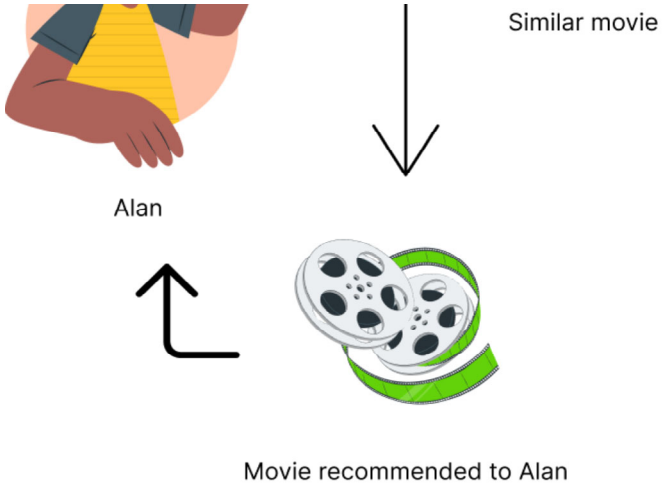
Of course, sparse matrices are not sufficient for training a model, and it is unreasonable to expect a complete dataset of explicit data. So, we learn the similarities a given user has with other users, and use these to predict how much they would like some *unseen* item.

While explicit feedback is preferable, it is also possible to use implicit feedback to reflect user behavior. Examples of implicit feedback include the browser history on a website, the number of clicks made on a given page, and a user's pattern of mouse movement. As opposed to explicit feedback, implicit feedback is represented by a densely filled matrix, because we (as researchers) can dictate how this data is to be collected or organized.

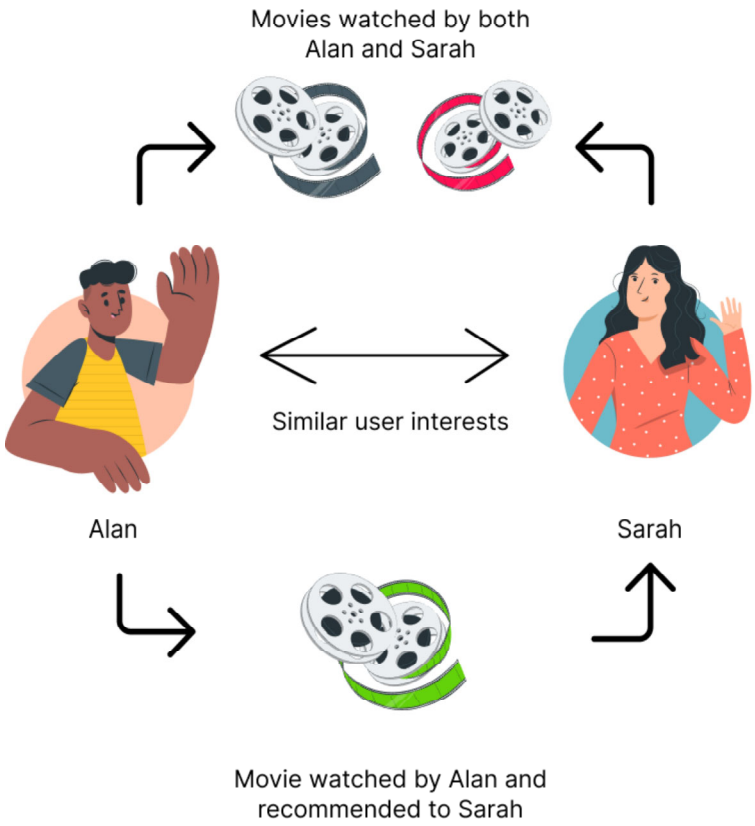
Classical recommender systems can be grouped into two main approaches: content-based (CB) and collaborative filtering (CF).



Content-based filtering generates a recommendation using additional information about the given user and item, through what we call *features*, which explain the observed interaction between a user and an item.



On the left is an illustration of the content-based process. If Alan has watched one movie, the recommender system will then find similar movies to it, and Alan will be given a recommendation from the set of movies most similar to the one he has already watched.



Collaborative filtering: generates a recommendation by relying on past user-item interaction, like explicit feedback (ratings) for movies watched in the past. In CF, it is sufficient to detect similar users and/or items, cluster them, and make new predictions based on the similarity existing within a cluster [6].

On the left is an illustration of the collaborative filtering process. If Alan and Sarah have watched the same movies and have the same interests, the recommender system will learn this information and recommend to Sarah a movie that Alan has watched.

The table below summarizes the advantages and disavantages of the two approaches. For more in-depth information, take a look at these references: [11, 12, 13].

	Content-based	Collaborative Filtering
Advantages	• Does not require any data about the other users • Can recommend items which are relevant to the user, but not the population (*"niche items"*)	• Does not require domain knowledge • Helps users discover new interests

Disadvantages	• Requires a lot of domain knowledge • Makes recommendatios based only on the existing interests of the user (no *"niche items"*)	• Cannot handle fresh items or users (the *"Cold-Start Problem"*)

Matrix Factorization: a collaborative filtering method

Matrix Factorization (MF) is a class of collaborative filtering techniques [3]. In its most simplistic implementa characterizes both the items and the users by a vector of factors (*matrices*), which then generate latent feat (*"embeddings"*) when they are multiplied together (*"into the matrix dot product"*).

This method has become popular because it offers flexibility for modeling real-life scenarios, while maintai scalability and predictive accuracy [2]. The toy example below includes only a very limited number of featu user and item matrices, allowing us to follow how predictions are computed, and understand how to interp that an individual feature may have on a given prediction. When this method scales up to a more realistic r features, the ability to interpret the prediction is lost [19]. There exists much academic debate on uncoverir "explainability of latent factor models". To learn more about it, see the following references [19, 20, 21, 22].

> ***This box is interactive!*** *Hover over the matrices below to learn how they are used to calculate the dot proc make a recommendation)*

User matrix

	f1	f2
A	0.3	0.6
B	0.5	0.4
C	0.2	0.7
D	0.6	0.1

•

Item matrix

	W	X	Y	Z
f1	3.5	3.3	2.3	1.3
f2	4.5	1.5	3.7	2.8

=

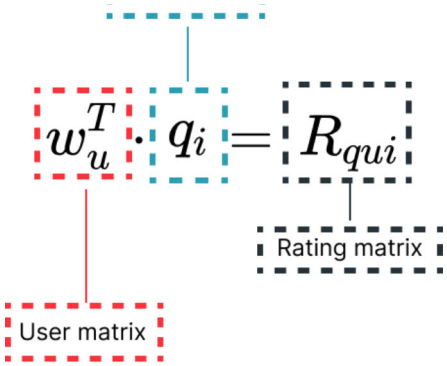
Rating matrix

User \ Item	W	X	Y
A	3.8	1.9	2.9
B	3.6	2.3	2.6
C	3.9	1.7	3.0
D	2.6	2.1	1.8

Each item i and user u are associated with a vector q_i and w_u, respectively. For a given item, the correspond measures the degree to which a feature represents the item. Similarly, the user vector w_u measures the deg the user has in the item.

The interaction, defined as "the interest of the user u in item i" , is then captured by the dot product of the Once all the dot products are computed, it becomes possible to rank the predicted ratings and identify the recommend to any given user.

Item matrix



Matrix Factorization formula.

This box is interactive! *Adjust your preferences through the movie genre sliders for comedy and horror, an on "Calculate Matrix Factorization" to see how your preferences align with those of your friends'*

Let's consider the following example: you are planning a movie night with your friends Anna, Jonny, and I know that Anna likes both horror movies and comedy, Jonny has a strong preference for comedy, and Ki horror movies, but also enjoys comedy once in a while. To find a movie that everyone will enjoy, you are a recommender system based on the Matrix Factorization techniques described above.

Comedy preference:

Horror preference:

User	Comedy	Horror
Anna	0.6	0.5
Jonny	0.7	0.1
Kimi	0.4	0.9
You	0.5	0.5

Item	Comedy	Horror
Zombieland	3	2
Modern Times	5	1
The Grudge	1	5

CALCULATE MATRIX FACTORIZATION

How to read the results*: to help you decide which movie to watch, look at the last row of the Matrix Factor results. This row represents each movie's average predictive rating within your group.*

While Matrix Factorization can produce good results in a short time, a naïve approch has a time complexity where *m* are the number of users, *n* the number of items, and *k* the number of latent factors (a hyperparam method has its disavantages:

- **Cold start problem**: The matrix cannot handle fresh items, such as new movies or new users the traini include [14].
- **Recommendation relevance**: Matrix Factorization uses the dot product to recommend items. If all use interacted and liked the same item, the recommendation will focus on that item.
- **Hard feature encoding**: In order to generate a recommendation, we have to explicitly provide the syst preferences and item features.

Embedding powered systems

Nowadays, recommender systems consider many more features than our MF example above:

- **Categorical**: userID, itemID, item brand, genre, language, etc.
- **Numerical**: price, delivery time, number of reviews, average of the reviews, etc.
- **Unstructured**: keywords, colors, material, review text, etc.

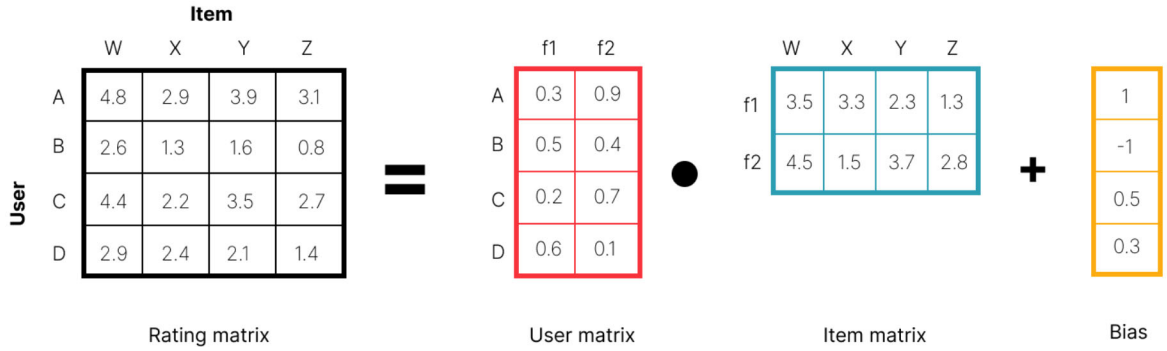
In a real-world scenario, we likely would not have explicit data on the preferences of every user and the fea item. This hinders our ability to make optimal recommendations. How might we resolve this?

EmbeddingMF: An embedding approach to matrix factorization

Building on top of our previous approach, Matrix Factorization, we learn the latent factors (implicit characte each movie and user, based on user-movie ratings. Learning these characteristics can be done through app gradient descent [16].

We will use the *"EmbeddingMF"* approach, which includes a general bias term on top of our existing dot pr EmbeddingMF is a simplification of the FunkSVD method. FunkSVD, unlike our EmbeddingMF model, incor least three separate bias terms (general, user, item), on top of the matrix factorization [2].

Note: Our inclusion of a single bias term serves to broadly illustrate to the reader the effect of bias on an RS r although EmbeddingMF is not the most effective or cutting-edge RS, it is sufficient for our purposes.



Augmented MF computation required for EmbeddingMF. Note the addition of the bias term (in yellow).

We start by creating two embedding matrices:

1. **A user embedding matrix *U***, containing one user per row and *n* user features, as columns.
   E.g. For a set of 100 users and 200 features, U will have dimensions of (100, 200)

2. **A movie embedding matrix *M***, containing one movie per row and *n* movie features, as columns.
   E.g. For a set of 100 movies and 200 features, M will have dimensions of (100, 200)

Having the same number of columns in each matrix makes the dimensions favourable for multiplying the t
matrices ($U \times M^T$). After multiplication, we add bias terms to each user and each movie. Adding bias to our
product results in a matrix representing predictive user ratings for all the movies in our dataset. Since these
(features) are initialized randomly (independently and identically distributed Normal samples, *i.i.d*), the initi
computed by the model will likely differ significantly from the ground truth. Through training, the differenc
latent factors and the ground truth is minimized.

The model consists of a user matrix of dimensions *(n_users x 128 features)*, a movie matrix of dimensions *(n*
features), a user bias vector of length *[n_users]*, and a movie bias vector *of length [n_movies]*. We randomly
latent factors for every user and movie with i.i.d samples from a Normal of mean 0 and standard deviation
the multiplication of users and movies, we apply a sigmoid range transformation that squeezes the results
between 0 and 5.5. We chose to use 5.5 so that the distribution of the error for those movies whose true ra
always negative. This allows the model to learn values only within this range for its predicted ratings. We tr
model over 15 epochs using the Mean Squared Error loss (MSELoss), a learning rate of 0.005, and a weight
[17]. Since these hyperparameters gave us sufficiently robust results, we did not tune them further.

EmbeddingMF was able to generate movie rating predictions with an average validation error of 0.71. The
to train and test the model included the MovieLens 1M columns of user ID, movie ID, and the movies' genr
MovieLens 1M also provides data on user occupations, ages, and locations (via US zip codes). How might v
metadata in improving our validation error?

DeepFM: A deep learning factorization machine

The adoption of deep learning models has been on the rise in every domain, and recommender systems ar
exception. In this section, we build upon Matrix Factorization, turning it into something we call a factorizati
(FM). More on that in a minute. The example we use to a factorization machine (FM)-based neural network
deep learning recommender system developed by Guo et. al. for Huawei in 2017 [6]. The DeepFM model h
that considers several concepts we have discussed above, combining these into a (then considered) novel r
architecture.

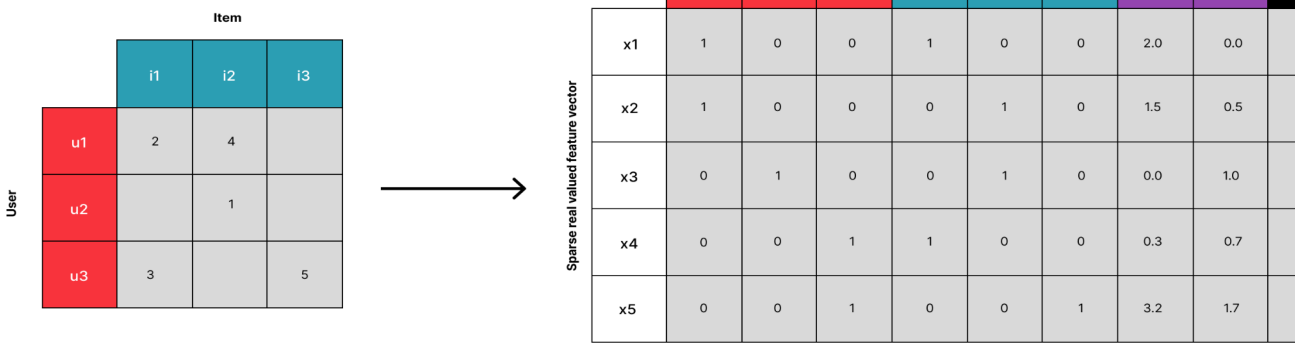
We haven't covered FMs yet-- this is coming up, don't fret!

DeepFM's output unit combines information from two components: a factorization machine, and a neural r
of these components accept a shared input of raw (sparse) feature data. The neural network learns dense e
from the sparse data, uncovering patterns pointing to both low- and high-order interactions between featu

low-order feature interactions like FM, and models high-order feature interactions like DNN. Due to the rel between the FM and NN, this model can learn recommendations without any additional feature engineerir accomodate input data which includes vectors of varying lengths. DeepFM's initial dense embedding layer the input to a low-dimensional, low-sparsity, real-value vector, before further this into the FM, and the first of the deep component. No matter how sparse our input data may be with respect to various features, Dee embeddings of the appropriate shape and appropriate sparsity for its components. We choose DeepFM fo recommendation system progression because machine factorization combines in its computation concepts matrix factorization with regression. When we consider that we began with a relatively simple matrix factor DeepFM can be seen as an "upgrade" to both the two models we explored above.

Architecturally, DeepFM consists of two components that share the same input: a Factorization Machine (FI neural network.

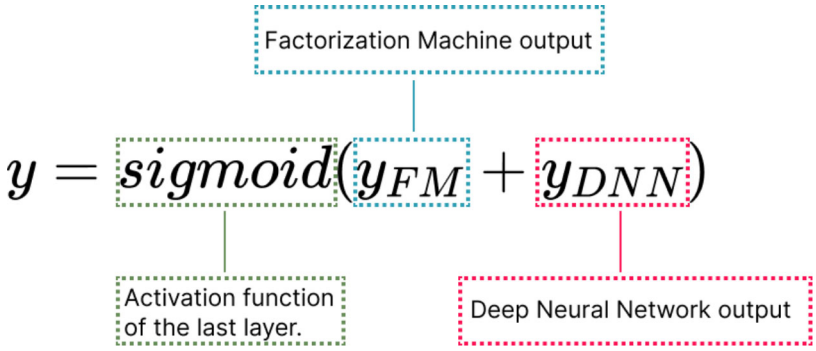
Factorization Machines (FM): A model class combining the advantages of Support Vector Machines with models [7]. FM model all interactions between variables using factorized parameters-- user-item interactio represented as tuples. These tuples are vectors consisting of real-valued and numeric variables that FM tre learning "*n-way*" interactions between them, n being the relevant degree of interaction for the design of sc model. The computation FM conducts includes a weight for each base feature, and an interaction term rep feature combinations [23]. Particularly useful in data contexts with high sparsity! Including the FM compon the overall DeepFM architecture, in addition to the DNN component, eliminates the need to pre-train the f allowing us to jointly train the combined model (FM + DNN) end-to-end. The outputs of the FM become la vectors, and used as network weights in the output units of the overall model (rather than a random initiali



represenation of Matrix Factorization which inputs are user-item interactions. On the left a Factorization Machine where interactions are represented by tuples of real-valued feature vectors and numeric target variables.

Deep component: The deep component is a feed-forward neural network, used to learn both low- and hig feature interactions. That means that, for a given feature i, a weight w_i represents its order-1 (linear) import latent vector V_i is used to measure the impact of interactions feature i has with other features the model cc latent vector V_i is then fed to the FM component, to model order-2 (pairwise) feature interactions. The resu fed to the deep component, to model the high-order feature interactions.

All parameters are trained jointly, rather than pre-training via the FM and then applying the DNN. The com prediction y is represented by the following formula:



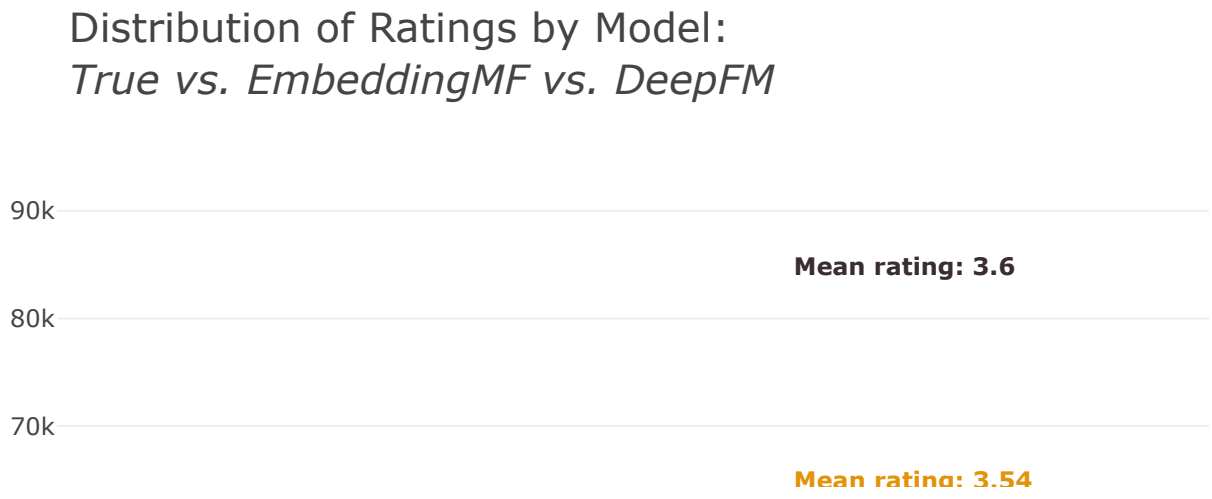
Mathematical formula to describe the output of the DeepFM model.

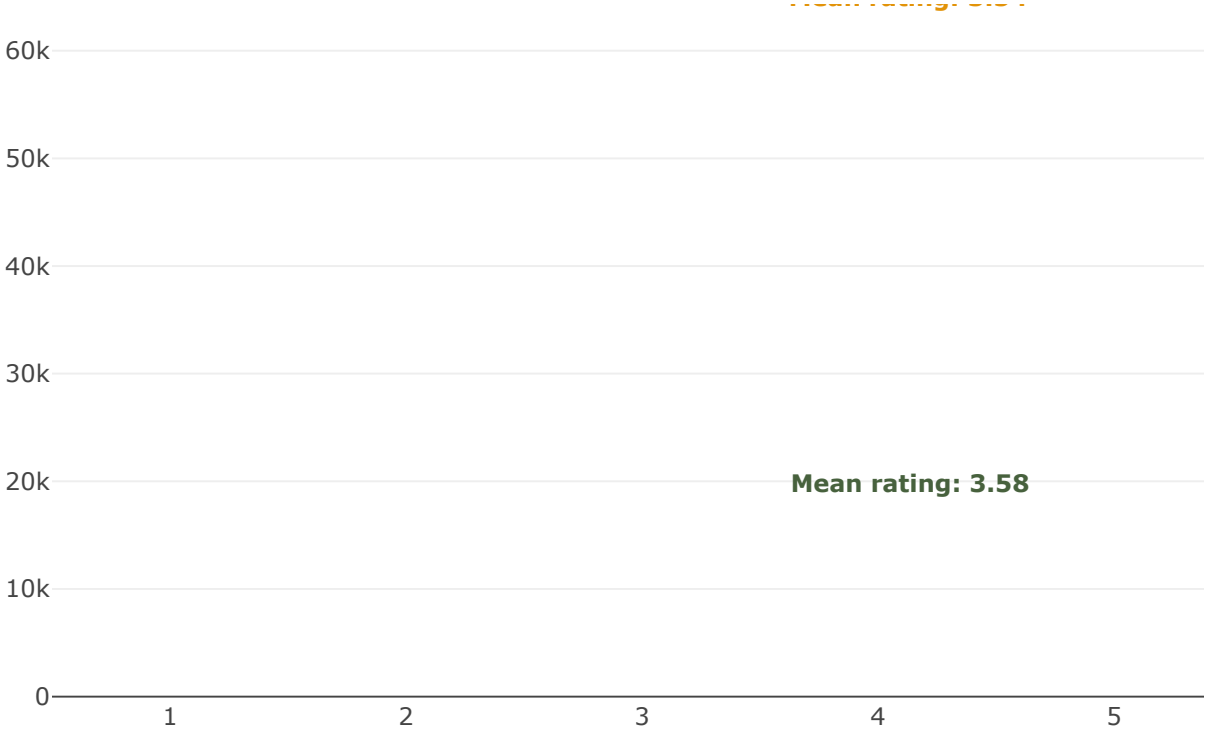
Our DeepFM model considers users, movies and their genres, and the users' occupation as an additional in included in EmbeddingMF. The FM outputs a latent vector of dimensions *(128,128)*, the same number of fe EmbeddingMF. We trained the model over 100 epochs, with a batch size of 2048, using MSELoss, and a dro 0.5 [6]. Our training of DeepFM concluded in a validation error of 0.7553, comparable to our implementatic EmbeddingMF.

One recommender cannot rule them all

> ***"All models are wrong, but some are useful."***
>
> ***- George Box, 1976, [15]***

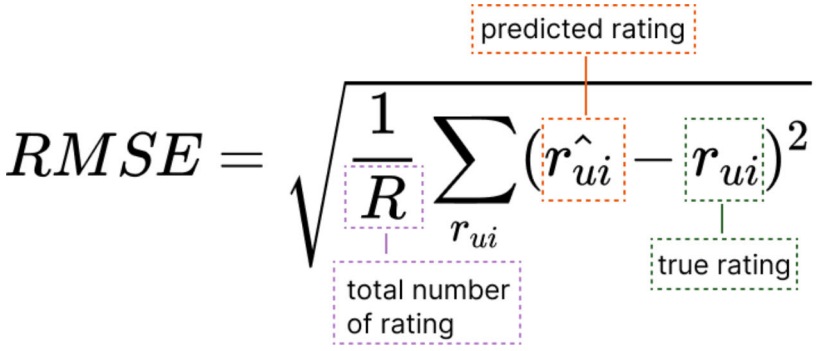
We have trained EmbeddingMF and the DeepFM for the task of generating recommendations with a train/ 900k/100k entries of users' movie ratings [18]. By plotting the distribution of the true and predictive rating note that that the distribution of the DeepFM is very narrow and highly dense around a rating of 3.2, while EmbeddingFM has a density focused around 3.7. What is surprising here are the mean predictive ratings, w quite differing density shapes are, in fact, very close to each other— and to the training mean.





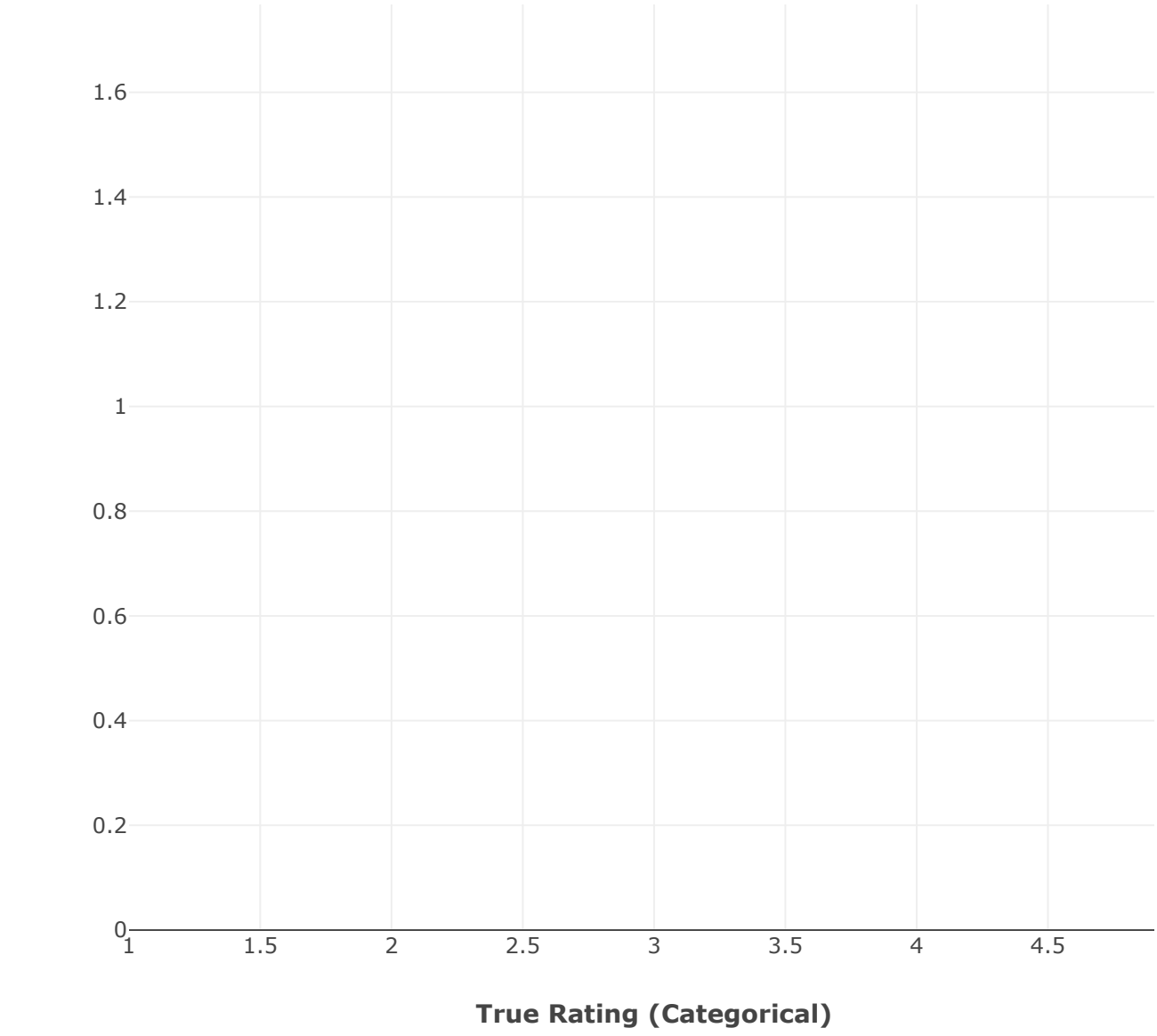
Distribution and mean of the predictive models' and training data's ratings across all movies

To better understand these results, we validate our models using the Root Mean Squared Error (RMSE), a st widley used for validation in the recommender systems research. We decides to use RMSE over other evalu like MAP or nDCG, for its understand semplicity. Moreover, RMSE is typically used when we want to evalua score, such as the predictive rating of a movie, and compare it to a ground truth (true rating) as we want to case. Compared to other metrics, RMSE penalizes more harshly a larger absolute error, giving you a measu concentrated your data is around the curve you have fit through your model. Lower RMSE translates to bet recommendation accuracy [1, 8].



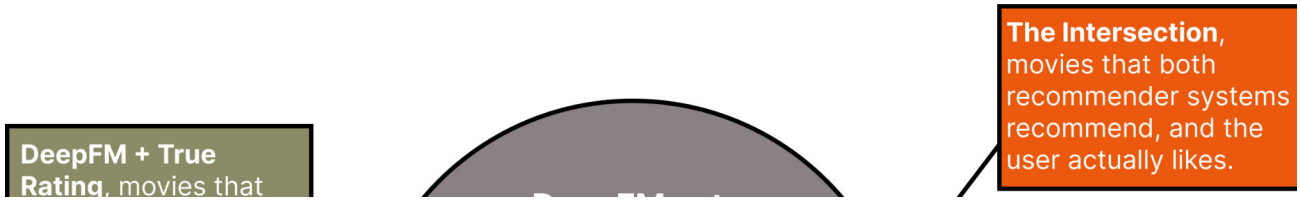
Root Mean Square Error formula.

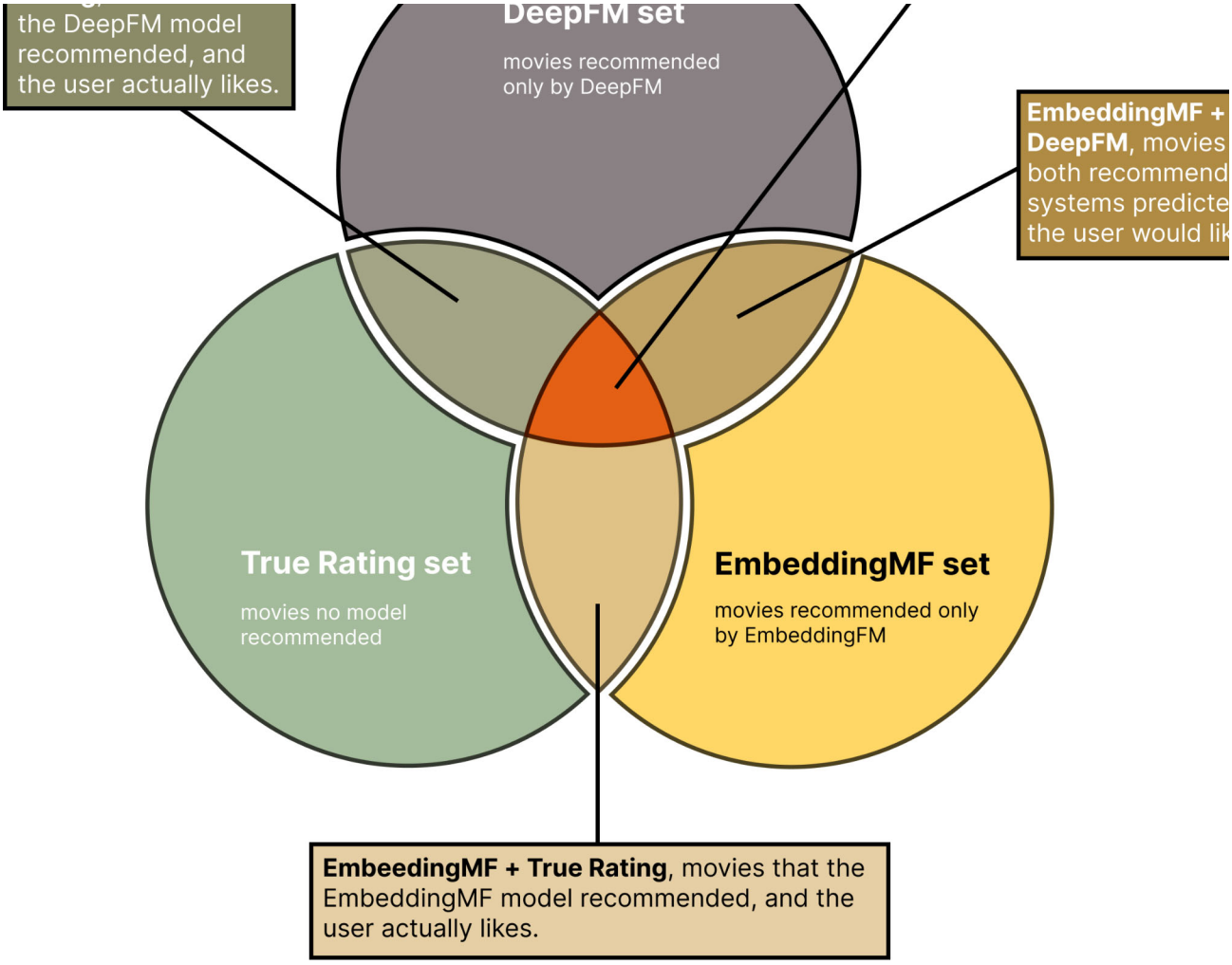
By plotting the RMSE of the two models we notice that both EmbeddingMF and DeepFM present a high RI the two extremes: predicted ratings of 1 to 2 and of 5. The error on the lower rating extreme might be expl considering that “people are more willing to share positive experiences than negative ones”, and so as a re train more around positive ratings than negative ones [9]. How might we explain the upper extreme?



RMSE of EmbeddingMF (yellow) and DeepFM (grey), both featuring large RMSE values at the two extremes of true rat

We define a perfect recommendation system as one which will recommend to users only movies they will b a rating of 5 (as a proxy for a high level of enjoyment of the recommendation). The above plot shows that systems we have experiemented with, while more competent than prediting the low rating of a movie, are perfect under our defined criterium. We dig deeper into this concerning finding by exploring how our two generated movie recommendations for Xiao, as example user. Explore with us by examining the Venn diagr



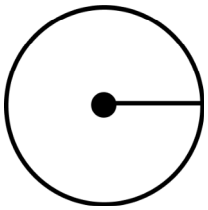


Description of the different areas of the Venn Diagram, which examines 3 sets of movies: true ratings (movies rated 5 b EmbeddingMF predicted ratings, and DeepFM predicted ratings.

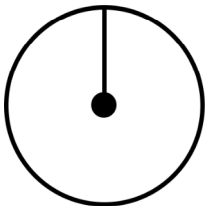
This box is interactive! *The movies are represented by dots in the different sets and intersections of the Ve The line that starts from the center of the circle represents the error (RMSE) that the model made in generat prediction. Hover over one a point to discover the movie's name, its true rating, and its predicted scores from*

Let's consider Xiao. He is a young writer, between 25-34 years old, living in the West Coast (USA). We war understand his preferences and what the two recommenders should, and have, recommended to him. To have retrieved the top 10 movies from both the recommendation models, and a set of movies that Xiao h ranked 5. We examine the intersections of these sets by visualizing them in a Venn diagram, as described annotated legend above.

EmbeddingFM

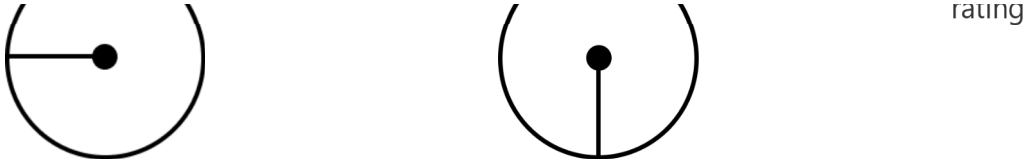


DeepFM



The model underestimatec user's rating

The model overestimated



Xiao's True and Predicted Ratings

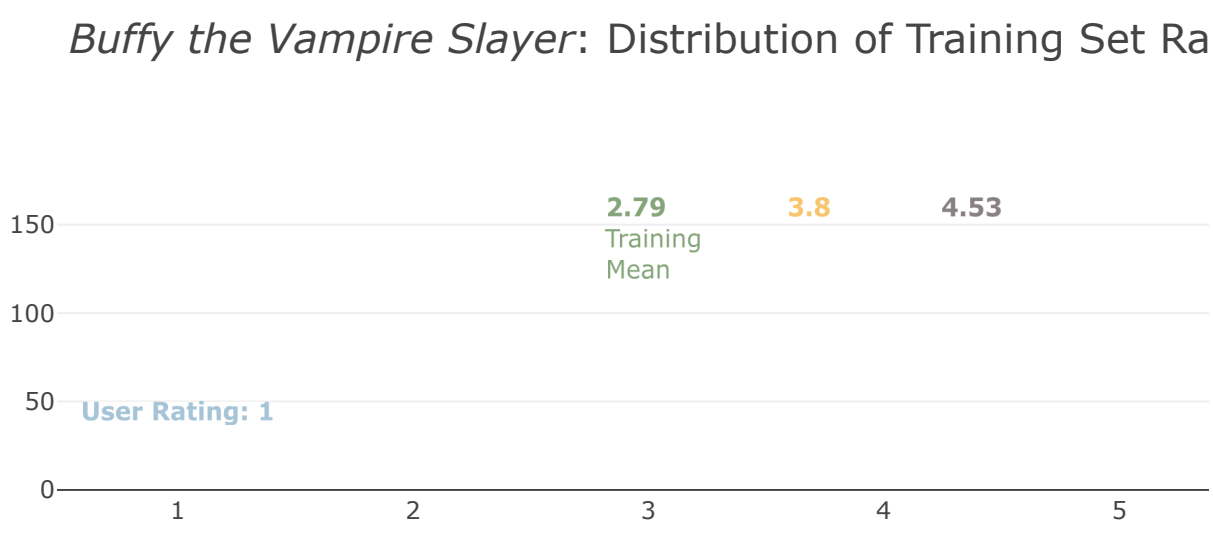
DeepFM

Movies rated 5 by Xiao

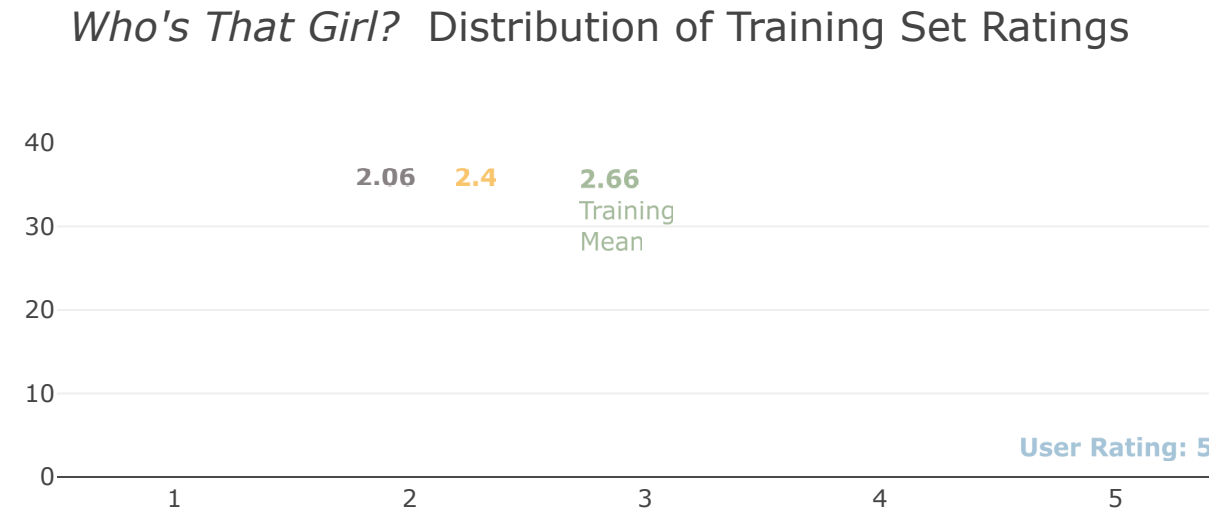
EmbeddingFM

Interesting to note in the above diagram are two movies with particularly large radii, located in the top left rating set, and in the intersection between the two models. The former movie is one that, in truth, the user rated 5, but neither of the models decided to recommend. The latter movie is one that both models predic user would like, but in truth, the user has rated 1! How did this happen?

We want to further explore these cases, where the movies recommended and the user preferences do not we can determine if any sort of bias might be plaguing our models.



Dashed lines in **grey (DeepFM)** and **yellow (EmbeddingMF)** reflect the prediction of each model for **Xiao (User**



Looking at these (true) ratings distributions, we notice a skew and an elongated tail to each histogram. In t
Buffy, the distribution leans left and the models' mean predictions are fairly high. Despite the fact that we k
hated *Buffy*, this suggests that, on average, users enjoyed it. *Who's That Girl?* shows us the opposite trend,
observation that the mean predicted ratings of EmbeddingMF and DeepFM deviate much less from the tra
mean rating, likely because of its substantially lower number of ratings (88 in total to *Buffy's* 465). What mi
caused this?

Maybe some characterists of Xiao's were overlooked, making him an outlier in these rating sets? Let us see
trends (for users' occupation, we exclude from consideration the MovieLens category 0: undefined):

BUFFY THE VAMPIRE SLAYER

- Reported gender breakdown: male 351, female 114
- Reported user occupations (top 3): 79 college/grad student, 36 executive/managerial, 34 technician/en
  writers like Xiao, we see 21 ratings
- Reported age brackets (top 3): 197 (25-34), 127 (18-24), 68 (35-44)

WHO'S THAT GIRL?

- Reported gender breakdown: male 52, female 36
- Reported user occupations (top 3): 15 college/grad student, 9 executive/managerial and sales/marketir
  academic/educator. For writers like Xiao, we see 3 ratings
- Reported age brackets (top 3): 45 (25-34), 20 (18-24), 17 (35-44)

Conclusion

Why didn't Xiao like the movies DeepFM and EmbeddingMF recommended to him? A system is rarely awa
does not know, and is limited by the data it is provided for training. What if these features, this metadata, a
correct sources of information for predicting what our user may like to watch? Xiao was so similar to the de
the users who enjoyed *Buffy the Vampire Slayer*, yet his true rating for this movie was 1. What aspects of *Bu*
responsible for Xiao's dislike?

How we might learn the answer to this question, and how we might collect data for recalibrating recomme
to specific user needs, are open questions in the field of recommender systems research. While recommen
aim to generate a personalized experience on the web, models are trained on a group of users with similar
not on a single user and their individual opinions. Through exploration of Xiao's top recommendations, we
the features we have used for characterizing user similarities may not be sufficient for making good recom
Such exploration has allowed us insight, beyond model fit and validation losses, into the fact that there are
out there, missing from our models, that can make our recommendations into the personalized experience

What do we ask users to learn these niche recommendations? How do we collect, or compute, such abstra
representations? The discovery of machine-recommended content that speaks to, and connects with us on
level may require moving beyond a *one-model-fits-all-users* approach. Calibrating our models per-user, thr
user feedback and desires, could be the key to reaching our personalized recommendation dreams— and t
answering that one *infuriating* question:

What should we watch tonight?

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