Efficient Algorithms for Frequently Asked Questions

5. Width Measures

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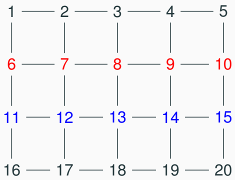
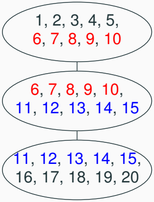
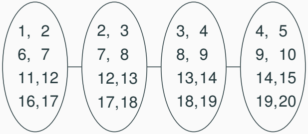
Choosing a Good Decomposition

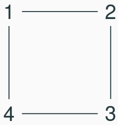
Hypergraphs

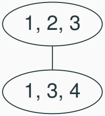
Choose the left or the right decomposition?











Choosing a Good Decomposition

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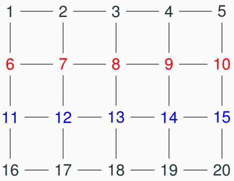

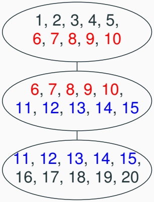

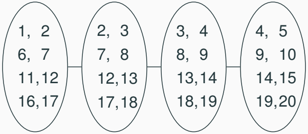

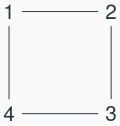

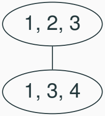

TL;DR: We need to measure the goodness of a hypertree decomposition

Overview of Width Measures Covered in This Lecture

We will look at a variety of measures of goodness typically called widths

- Treewidth $\mathtt{tw}$
- Edge cover number ρ and fractional edge cover number ρ^*
- Hypertree width $\mathtt{hw}$ and fractional hypertree width $\mathtt{fhw}$

$$\rho(\mathcal{H}) \underbrace{\geq}_{|\mathcal{E}|-1} \mathtt{hw}(\mathcal{H}) \qquad \text{and} \qquad \rho^*(\mathcal{H}) \underbrace{\geq}_{|\mathcal{E}|-1} \mathtt{fhw}(\mathcal{H})$$

Given FAQ Φ with hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ and factors of size N

- The time to compute Φ is $O(N^{\mathtt{w}(\mathcal{H})})$, where
- the width $\mathtt{w}$ is any of: ρ; ρ^*; $\mathtt{hw}$; $\mathtt{fhw}$; $\mathtt{tw} + 1$

Treewidth: Counting the Number of Vertices in a Bag

Given: Hypergraph $\mathcal{H}$ with set $\mathbf{T}(\mathcal{H})$ of decompositions

- Treewidth $\texttt{tw}$ of hypertree decomposition $\mathcal{T} = (T, \chi)$:

$$\texttt{tw}(\mathcal{T}) = \max_{t \in T} |\chi(t)| - 1$$

  In words: The size of the largest bag in $\mathcal{T}$ minus one

- Treewidth $\texttt{tw}(\mathcal{H})$ of hypergraph $\mathcal{H}$:

$$\texttt{tw}(\mathcal{H}) = \min_{\mathcal{T} \in \mathbf{T}(\mathcal{H})} \texttt{tw}(\mathcal{T})$$

  In words: The smallest treewidth of any of its decompositions

The computation time is $O(N^{\texttt{tw}(\mathcal{H})+1})$, where N is max domain size for variables

Quiz: What is the treewidth for the grid and 4-cycle hypergraphs?

Width Measures Based on Factor Sizes

Use factor sizes instead of domain sizes

- Treewidth assumes factors $\approx$ the products of the domains of their variables
- However, factors are typically much smaller than the products of the domains of their variables in many domains (e.g., DB, CSP, SAT)

Measures Based on Factor Sizes: The Triangle Example

Triangle query: $\Phi(x_1, x_2, x_3) = \psi_{12}(x_1, x_2) \otimes \psi_{23}(x_2, x_3) \otimes \psi_{13}(x_1, x_3)$

- Assumption: $|\psi_{ij}| < N$
- Upper bound on computation time: $|\psi_{12}| \cdot |\psi_{23}| = N^2$
  - Iterate over all combinations of tuples in ψ_{12} and ψ_{23}
  - Ensure the combinations use the same x_2 in both ψ_{12} and ψ_{23} and also use (x_1, x_3) that occur in ψ_{13}

Measures Based on Factor Sizes: The Triangle Example

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  - Ensure the combinations use the same x_2 in both ψ_{12} and ψ_{23} and also use (x_1, x_3) that occur in ψ_{13}
- Lower bound: There are factors for which the computation time is at least N
  - $\psi_{12} = \{1\} \times [N]$, $\psi_{23} = [N] \times \{1\}$, $\psi_{13} \supseteq \{(1, 1)\}$

Can we close the gap between lower and upper bounds on computation time?

Edge Covers and Independent Sets

We can generalise the analysis of the triangle query

For the upper bound:

- Cover all nodes (variables) by k edges (factors) $\Rightarrow$ size $\leq N^k$.
- Pick the smallest k so to reduce the complexity
- The smallest k is the edge cover number denoted ρ
- This is an edge cover of the query hypergraph!

For the lower bound:

- m independent nodes (variables) $\Rightarrow$ construct factors such that size $\geq N^m$.
- Pick the largest m so to get a realistic/higher lower bound on the complexity
- This is an independent set of the query hypergraph!

Edge Covers and Independent Sets Expressed using Integer Programming

Minimum Edge Cover of hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$:

$$\begin{aligned} \text{minimise} \quad & \textstyle\sum_{S \in \mathcal{E}} w_S \\ \text{subject to} \quad & \textstyle\sum_{S \in \mathcal{E} : v \in S} w_S \geq 1 \;\; \forall v \in \mathcal{V}, \\ & w_S \in \{0, 1\} \qquad \forall S \in \mathcal{E} \end{aligned}$$

The cost of an optimal feasible solution is the edge cover number ρ

Maximum Independent Set of hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$:

$$\begin{aligned} \text{maximise} \quad & \textstyle\sum_{v \in \mathcal{V}} w_v \\ \text{subject to} \quad & w_{v_1} + w_{v_2} \leq 1 \;\; \forall v_1, v_2 \in S, S \in \mathcal{E} \\ & w_v \in \{0, 1\} \qquad \forall v \in \mathcal{V} \end{aligned}$$

The costs of feasible solutions for the two optimisation programs may differ

Fractional Edge Covers and Fractional Independent Sets

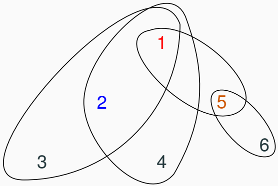
Relaxation of the two optimisation problems as linear programs:

- Variables can now range between 0 and 1 instead of being either 0 or 1
- By duality of linear programming, the two linear programs have the same cost of feasible solutions
- This cost $\rho^*(\mathcal{H})$ is called the fractional edge cover number

$$
\begin{aligned}
\text{minimise} \quad & \textstyle\sum_{S \in \mathcal{E}} w_S \\
\text{subject to} \quad & \textstyle\sum_{S \in \mathcal{E} : v \in S} w_S \geq 1 \quad \forall v \in \mathcal{V}, \\
& 0 \leq w_S \leq 1 \qquad\qquad \forall S \in \mathcal{E}
\end{aligned}
$$

Example (1/3): Fractional Edge Cover for an Acyclic Hypergraph

$$\Phi(x_1, \ldots, x_6) = \psi_{123}(x_1, x_2, x_3) \otimes \psi_{124}(x_1, x_2, x_4) \otimes \psi_{15}(x_1, x_5) \otimes \psi_{56}(x_5, x_6)$$



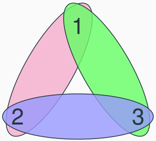
- The three edges (factors) $\psi_{123}, \psi_{124}, \psi_{56}$ can cover all nodes (variables)
$$\rho^*(\mathcal{H}) = \rho(\mathcal{H}) \leq 3$$
- Each node (variable) 3, 4, and 6 must be covered by a distinct edge (factor)
$$\text{(Fractional)IndependentSet}(\mathcal{H}) \geq 3$$

$\Rightarrow \rho^*(\mathcal{H}) = \rho(\mathcal{H}) = 3$

$\Rightarrow$ For input size N, Φ takes time $O(N^3)$ and for some inputs this is $\Theta(N^3)$

Example (2/3): Fractional Edge Cover for the Triangle Hypergraph

$$\Phi(x_1, x_2, x_3) = \psi_{12}(x_1, x_2) \otimes \psi_{23}(x_2, x_3) \otimes \psi_{13}(x_1, x_3)$$



$$\begin{array}{lcccccl}
\multicolumn{7}{l}{\textit{minimise } w_{\psi_{12}} + w_{\psi_{13}} + w_{\psi_{23}}} \\
\multicolumn{7}{l}{\textit{subject to}} \\
1: & w_{\psi_{12}} & + & w_{\psi_{13}} & & & \geq 1 \\
2: & w_{\psi_{12}} & & & + & w_{\psi_{23}} & \geq 1 \\
3: & & & w_{\psi_{13}} & + & w_{\psi_{23}} & \geq 1 \\
 & w_{\psi_{12}} \geq 0 & & w_{\psi_{13}} \geq 0 & & w_{\psi_{23}} \geq 0 &
\end{array}$$

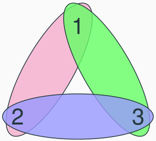
Our previous upper bound was $O(N^2)$, since the edge cover number is 2:

- Set any two of $w_{\psi_{12}}, w_{\psi_{13}}, w_{\psi_{23}}$ to 1

What is the fractional edge cover number for the triangle hypergraph $\mathcal{H}$?

Example (2/3): Fractional Edge Cover for the Triangle Hypergraph

$$\Phi(x_1, x_2, x_3) = \psi_{12}(x_1, x_2) \otimes \psi_{23}(x_2, x_3) \otimes \psi_{13}(x_1, x_3)$$



$$\begin{array}{lllllll}
\textit{minimise } w_{\psi_{12}} + w_{\psi_{13}} + w_{\psi_{23}} \\
\textit{subject to} \\
1: & w_{\psi_{12}} & + & w_{\psi_{13}} & & & \geq 1 \\
2: & w_{\psi_{12}} & & & + & w_{\psi_{23}} & \geq 1 \\
3: & & & w_{\psi_{13}} & + & w_{\psi_{23}} & \geq 1 \\
 & w_{\psi_{12}} \geq 0 & & w_{\psi_{13}} \geq 0 & & w_{\psi_{23}} \geq 0 &
\end{array}$$

Our previous upper bound was $O(N^2)$, since the edge cover number is 2:

- Set any two of $w_{\psi_{12}}, w_{\psi_{13}}, w_{\psi_{23}}$ to 1

What is the fractional edge cover number for the triangle hypergraph $\mathcal{H}$?

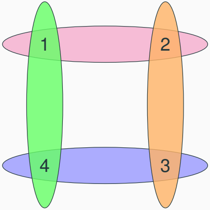
We can do better: $w_{\psi_{12}} = w_{\psi_{13}} = w_{\psi_{23}} = 1/2$. Then, $\rho^*(\mathcal{H}) = 3/2$

Lower bound reaches $N^{3/2}$ for $\psi_{12} = \psi_{13} = \psi_{23} = [N^{1/2}] \times [N^{1/2}]$

Example (3/3): Fractional Edge Cover for 4-Cycle Hypergraph

$$\Phi(x_1, \ldots, x_4) = \psi_{12}(x_1, x_2) \otimes \psi_{23}(x_2, x_3) \otimes \psi_{34}(x_3, x_4) \otimes \psi_{14}(x_1, x_4)$$

The linear program for the fractional edge cover number:



$$\begin{array}{lllllll}
\textit{minimise } w_{\psi_{12}} + w_{\psi_{23}} + w_{\psi_{34}} + w_{\psi_{14}} \\
\textit{subject to} \\
1: & w_{\psi_{12}} & & & + & w_{\psi_{14}} & \geq 1 \\
2: & w_{\psi_{12}} & + & w_{\psi_{23}} & & & \geq 1 \\
3: & & & w_{\psi_{23}} & + & w_{\psi_{34}} & \geq 1 \\
4: & & & w_{\psi_{34}} & + & w_{\psi_{14}} & \geq 1 \\
& w_{\psi_{12}} \geq 0 & & w_{\psi_{23}} \geq 0 & & w_{\psi_{34}} \geq 0 \quad w_{\psi_{14}} \geq 0
\end{array}$$

Solution: $w_{\psi_{12}} = w_{\psi_{34}} = 1$. Alternative: $w_{\psi_{23}} = w_{\psi_{14}} = 1$. Then, $\rho^*(\mathcal{H}) = 2$.

Lower bound reaches N^2 for $\psi_{12} = \psi_{34} = \psi_{14} = [N] \times \{1\}$ and $\psi_{23} = \{1\} \times [N]$.

Refinement under Cardinality Constraints

Recall the linear program for computing the fractional edge cover number ρ^*

$$\text{minimise} \quad \sum_{S \in \mathcal{E}} w_S$$

$$\text{subject to} \quad \sum_{S \in \mathcal{E}: v \in S} w_S \geq 1 \ \forall v \in \mathcal{V},$$

$$0 \leq w_S \leq 1 \qquad \forall S \in \mathcal{E}$$

Equivalent formulation of minimisation objective assuming each factor has size N:

$$N^{\sum_{S \in \mathcal{E}} w_S} = \prod_{S \in \mathcal{E}} N^{w_S}$$

[We show here the refinement for ρ^*; it works similarly for ρ]

Refinement under Cardinality Constraints

Recall the linear program for computing the fractional edge cover number ρ^*

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Equivalent formulation of minimisation objective assuming each factor has size N:

$$N^{\sum_{S \in \mathcal{E}} w_S} = \prod_{S \in \mathcal{E}} N^{w_S}$$

In practice, factors have different sizes, i.e., ψ_S has size N_S:

$$\prod_{S \in \mathcal{E}} N_S^{w_S} \qquad \text{or by taking the log} \qquad \sum_{S \in \mathcal{E}} w_S \log N_S$$

[We show here the refinement for ρ^*; it works similarly for ρ]

Hypertree Width and Fractional Hypertree Width

- So far, we applied ρ or ρ^* to the entire hypergraph $\mathcal{H}$ of an FAQ
- We can also apply them to each bag of a hypertree decomposition $\mathcal{T}$ of $\mathcal{H}$
- This leads to (fractional) hypertree width of $\mathcal{T} = (T, \chi)$ and of $\mathcal{H}$

$$\mathtt{htw}(\mathcal{T}) = \max_{t \in T} \rho(\chi(t)) \qquad \text{and} \qquad \mathtt{fhtw}(\mathcal{T}) = \max_{t \in T} \rho^*(\chi(t))$$

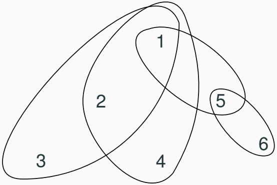
In words: The (fractional) hypertree width of a hypertree decomposition is the maximum (fractional) edge cover number of any of its bags

$$\mathtt{htw}(\mathcal{H}) = \min_{\mathcal{T} \in \mathbf{T}(\mathcal{H})} \mathtt{htw}(\mathcal{T}) \qquad \text{and} \qquad \mathtt{fhtw}(\mathcal{H}) = \min_{\mathcal{T} \in \mathbf{T}(\mathcal{H})} \mathtt{fhtw}(\mathcal{T})$$

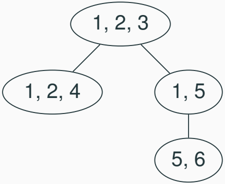
In words: The (fractional) hypertree width of a hypergraph is the minimum (fractional) hypertree width of any of its hypertree decompositions

Examples (1/3): (Fractional) Hypertree Width for Acyclic Hypergraph

α-acyclic hypergraph



hypertree decomposition

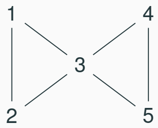


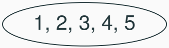
- Each bag of the decomposition only has variables from one hyperedge
- $\Rightarrow$ the (fractional) edge cover number is 1 for each bag
- $\Rightarrow$ the (fractional) hypertree width is 1 for the decomposition
- $\Rightarrow$ the (fractional) hypertree width is 1 for the hypergraph

The (fractional) hypertree width of any α-acyclic hypergraph is one

Examples (2/3): Hypertree Width for Bowtie Hypergraph

Consider the bowtie hypergraph $\mathcal{H}$ and two possible decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$





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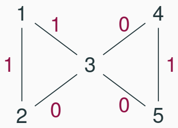

Left hypertree decomposition $\mathcal{T}_1$ is one bag for the entire hypergraph $\mathcal{H}$

- With the above weight assignments: $\rho(\mathcal{H}) = 3$
- $\texttt{htw}(\mathcal{T}_1) = \rho(\mathcal{H}) = 3$, since we only have one bag

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Consider the bowtie hypergraph $\mathcal{H}$ and two possible decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$

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- With the above weight assignments: $\rho(\mathcal{H}) = 3$
- $\texttt{htw}(\mathcal{T}_1) = \rho(\mathcal{H}) = 3$, since we only have one bag

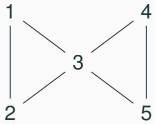
Right hypertree decomposition $\mathcal{T}_2$ has one bag per triangle in the hypergraph $\mathcal{H}$

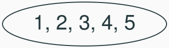
- We treat each bag independently: $\rho(\{1,2,3\}) = 2$, $\rho(\{3,4,5\}) = 2$
- $\texttt{htw}(\mathcal{T}_2) = \max\{\rho(\{1,2,3\}), \rho(\{3,4,5\})\} = 2$

Overall, $\texttt{htw}(\mathcal{H}) = \texttt{htw}(\mathcal{T}_2) = 2$, while $\rho(\mathcal{H}) = 3$

Examples (2/3): Fractional Hypertree Width for Bowtie Hypergraph

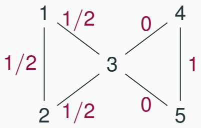
Consider again the bowtie hypergraph $\mathcal{H}$ and two decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$





Examples (2/3): Fractional Hypertree Width for Bowtie Hypergraph

Consider again the bowtie hypergraph $\mathcal{H}$ and two decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$



Left hypertree decomposition $\mathcal{T}_1$ is one bag for the entire hypergraph $\mathcal{H}$

- With the above weight assignments: $\rho^*(\mathcal{H}) = 5/2$
- $\texttt{fhtw}(\mathcal{T}_1) = \rho^*(\mathcal{H}) = 5/2$, since we only have one bag

Examples (2/3): Fractional Hypertree Width for Bowtie Hypergraph

Consider again the bowtie hypergraph $\mathcal{H}$ and two decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$

Left hypertree decomposition $\mathcal{T}_1$ is one bag for the entire hypergraph $\mathcal{H}$

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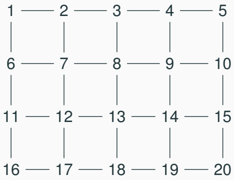
Right hypertree decomposition $\mathcal{T}_2$ has one bag per triangle in the hypergraph $\mathcal{H}$

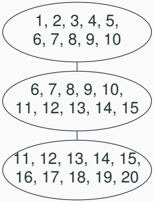
- We treat each bag independently: $\rho^*(\{1,2,3\}) = 3/2$, $\rho^*(\{3,4,5\}) = 3/2$
- $\texttt{fhtw}(\mathcal{T}_2) = \max\{\rho^*(\{1,2,3\}), \rho^*(\{3,4,5\})\} = 3/2$

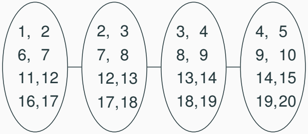
Overall, $\texttt{fhtw}(\mathcal{H}) = \texttt{fhtw}(\mathcal{T}_2) = 3/2$, while $\rho^*(\mathcal{H}) = 5/2$

Examples (3/3): (Fractional) Hypertree Width for the Grid Hypergraph

Consider the grid hypergraph $\mathcal{H}$ and two possible decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$







Examples (3/3): (Fractional) Hypertree Width for the Grid Hypergraph

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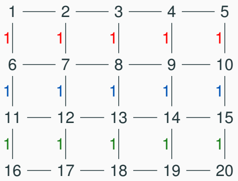
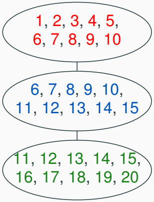
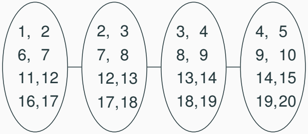




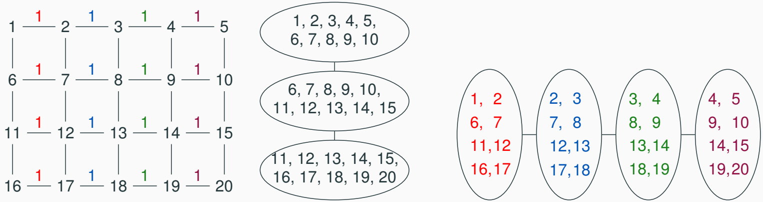


Left hypertree decomposition $\mathcal{T}_1$ has one bag for each two consecutive rows

- With the above weight assignments: $\rho^*(b) = \rho(b) = 5$ for each bag b in $\mathcal{T}_1$
- This means $\texttt{htw}(\mathcal{T}_1) \leq 5$ and $\texttt{fhtw}(\mathcal{T}_1) \leq 5$

Examples (3/3): (Fractional) Hypertree Width for the Grid Hypergraph

Consider the grid hypergraph $\mathcal{H}$ and two possible decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$



Left hypertree decomposition $\mathcal{T}_1$ has one bag for each two consecutive rows

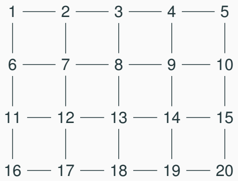
- With the above weight assignments: $\rho^*(b) = \rho(b) = 5$ for each bag b in $\mathcal{T}_1$
- This means $\mathtt{htw}(\mathcal{T}_1) \leq 5$ and $\mathtt{fhtw}(\mathcal{T}_1) \leq 5$

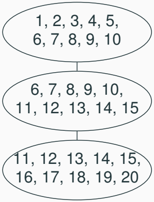
Right hypertree decomposition $\mathcal{T}_2$ has one bag for each two consecutive columns

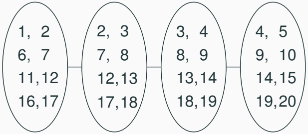
- With the above weight assignments: $\rho^*(b) = \rho(b) = 4$ for each bag b in $\mathcal{T}_2$
- This means $\mathtt{htw}(\mathcal{T}_2) \leq 4$ and $\mathtt{fhtw}(\mathcal{T}_2) \leq 4$

Examples (3/3): (Fractional) Hypertree Width for the Grid Hypergraph

Consider the grid hypergraph $\mathcal{H}$ and two possible decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$







Left hypertree decomposition $\mathcal{T}_1$ has one bag for each two consecutive rows

- With the above weight assignments: $\rho^*(b) = \rho(b) = 5$ for each bag b in $\mathcal{T}_1$
- This means $\texttt{htw}(\mathcal{T}_1) \leq 5$ and $\texttt{fhtw}(\mathcal{T}_1) \leq 5$

Right hypertree decomposition $\mathcal{T}_2$ has one bag for each two consecutive columns

- With the above weight assignments: $\rho^*(b) = \rho(b) = 4$ for each bag b in $\mathcal{T}_2$
- This means $\texttt{htw}(\mathcal{T}_2) \leq 4$ and $\texttt{fhtw}(\mathcal{T}_2) \leq 4$

Overall, $\texttt{htw}(\mathcal{H}) \leq \texttt{htw}(\mathcal{T}_2) \leq 4$ and $\texttt{fhtw}(\mathcal{H}) \leq \texttt{fhtw}(\mathcal{T}_2) \leq 4$

(Fractional) Hypertree Width in the Presence of Arbitrary Free Variables

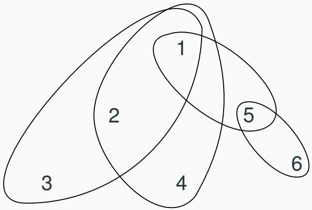
- Previous slides: Hypergraphs for FAQs without free variables
- Consider now: FAQs with hypergraph $\mathcal{H}$ and free variables $[f]$
- Free-connex property: The hypertree decomposition has a connected subtree that consists of all free variables $[f]$ and no bound variables
- Let $\mathbf{T}_{[f]}(\mathcal{H}) \subseteq \mathbf{T}(\mathcal{H})$ be the set of hypertree decompositions of $\mathcal{H}$ that satisfy the free-connex property for free variables $[f]$
- The (fractional) hypertree width of hypergraph $\mathcal{H}$ and free variables $[f]$

$$\mathtt{htw}(\mathcal{H}, [f]) = \min_{\mathcal{T}_{[f]} \in \mathbf{T}_{[f]}(\mathcal{H})} \mathtt{htw}(\mathcal{T}_{[f]}) \quad \text{and} \quad \mathtt{fhtw}(\mathcal{H}, [f]) = \min_{\mathcal{T}_{[f]} \in \mathbf{T}_{[f]}(\mathcal{H})} \mathtt{fhtw}(\mathcal{T}_{[f]})$$

Example: Hypertree Width for Acyclic Hypergraph with Free Variables

α-acyclic hypergraph $\mathcal{H}$

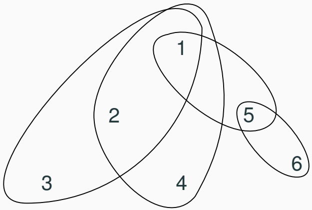
all/no variables free



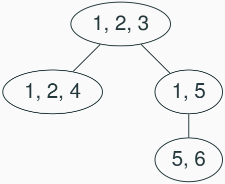
Example: Hypertree Width for Acyclic Hypergraph with Free Variables

α-acyclic hypergraph $\mathcal{H}$

all/no variables free



Possible join tree for $\mathcal{H}$

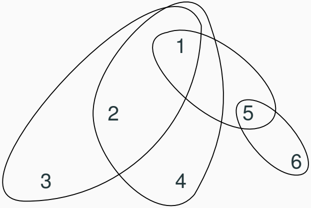


$\mathtt{htw}(\mathcal{T}) = \mathtt{fhtw}(\mathcal{T}) = 1$

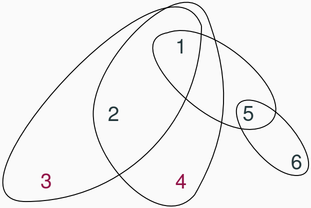
Example: Hypertree Width for Acyclic Hypergraph with Free Variables

α-acyclic hypergraph $\mathcal{H}$

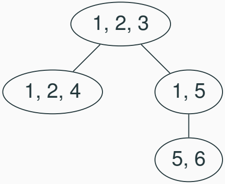
all/no variables free



choose $\{3, 4\}$
as free variables



Possible join tree for $\mathcal{H}$

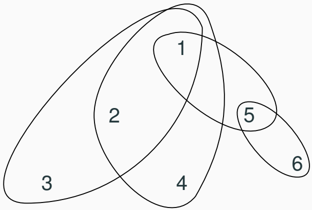


$$\mathtt{htw}(\mathcal{T}) = \mathtt{fhtw}(\mathcal{T}) = 1$$

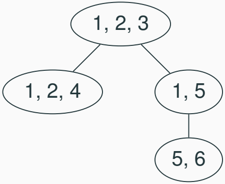
Example: Hypertree Width for Acyclic Hypergraph with Free Variables

α-acyclic hypergraph $\mathcal{H}$

all/no variables free

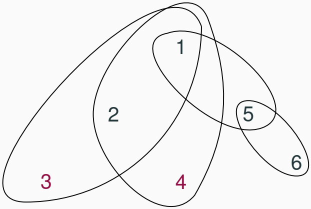


Possible join tree for $\mathcal{H}$

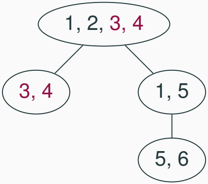


$\mathtt{htw}(\mathcal{T}) = \mathtt{fhtw}(\mathcal{T}) = 1$

choose $\{3, 4\}$
as free variables



Possible decomposition



$\mathtt{htw}(\mathcal{T}_{3,4}) = \mathtt{fhtw}(\mathcal{T}_{3,4}) = 2$