Computation and Economics - Spring 2012
Homework Assignment #08: Social Choice, Reputation Systems and Truthful Elicitation

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Out Thursday, May 3, 2012
Due **12:15** sharp: **Monday, May 14, 2012** (in class)
We strongly encourage typed submissions.

[**BSc 100 Points, MSc 120 Points**] This is a single-person assignment. Points will be awarded for clarity, correctness and completeness of the answers. Reasoning must be provided with every answer. You are free to discuss the problem set with other students. However, you should not share answers. **Copying will be penalized**.

1. [**31 Points**] Social Choice

   (a) [**8 Points**] Using an example, explain what it would mean for a social ranking function to fail the IIA property.

   (b) [**8 Points**] In the setup of Figure 12.1, identify the Dodgson winner by calculating the Dodgson score of each alternative.

   (c) [**5 Points**] State the Kemeny rule in your own words. Then show that in the case of only two alternatives, it is equivalent to the majority rule.

   (d) [**10 Points**] Consider the following preference profile

$$P = \begin{cases} 7: & a \succ b \succ c, \\ 2: & b \succ a \succ c, \\ 3: & b \succ c \succ a, \\ 8: & c \succ a \succ b. \end{cases}$$

   Calculate the result when using the Kemeny rule and when using the Schulze rule. Show all work. *(Hint: The outcome is the same from both rules.)*

2. [**31 Points, MSC + 5 Points**] Proper Scoring Rules

   (a) [**10 Points**] Consider the following setup:

   - There are two possible outcomes "sun" and "rain".
   - Two agents have different subjective beliefs about the probability of each event, given by the following table:

Agent	sun	rain
1	0.8	0.2
2	0.6	0.4

- Each agent is asked to report their estimate $\hat{p}$ of the probability for the event "sun".
- Each agent is rewarded based on a scoring rule after the realization of the event is observed.

i. **[5 Points]** Given a quadratic scoring rule, produce a plot of the agents' expected score (y-axis) over the reported parameter $\hat{p}$ (x-axis).

ii. **[5 Points]** Produce the same plot if a logarithmic scoring rule is used.

(b) **[5 Points]** How do theses plots support the fact that both scoring rules are proper? Also comment on the differences between the plots in i. and ii..

(c) **[8 Points]** Prove for a binary event that the quadratic scoring rule is proper, i.e. that the agent maximizes his score by reporting truthfully.

(d) **[MSc 5 Points]** Prove Proposition 14.1 of the class notes, i.e. show that if a scoring rule R is proper and $\alpha, \beta \in \mathbb{R}$ with $\beta > 0$, then $\tilde{R} := \alpha + \beta R$ is again proper.

(e) **[8 Points]** Suppose reports for the probability of a binary event can have the form $0.01 \cdot k, k \in \{1, \ldots, 99\}$, and in particular the smallest possible report is 0.01 and the largest is 0.99. Construct a proper scoring rule with positive payoff between 0 and 1, based on the logarithmic scoring rule.

3. **[21 Points]** Peer prediction

Consider the following peer prediction setting, where the quality of a TV set shall be assessed by two peers:

- The TV set has either a high quality (H) or a low quality (L)
- By examining the TV set, each agent reveives a private signal $S_i, i = 1, 2$ in $\{l, h\}$
- The prior is

$$P(L) = P(H) = 0.5,$$

  and the signal distribution is

$$P(h|H) = 0.6, P(l|L) = 0.8,$$

  and it is common knowledge to the agents and the mechanism
- The mechanism collects a report $\hat{s}_i, i = 1, 2$ from the agents
- The mechanism uses the reports to compute $g(\hat{s}_j|\hat{s}_i) = P(S_j = \hat{s}_j|S_i = \hat{s}_i), i \neq j$, the conditional probability of j observing signal $\hat{s}_j$, given i observes signal $\hat{s}_i$
- Agent i is scored using the logarithmic scoring rule, i.e. $R(\hat{s}_i, \hat{s}_j) := \ln(g(\hat{s}_j|\hat{s}_i)), i \neq j$.

(a) **[8 Points]** What are the conditional probabilities that agent 2 receives signal h, i.e. $g(h|h) = P(S_2 = h|S_1 = h)$ and $g(h|l) = P(S_2 = h|S_1 = l)$?

(b) **[8 Points]** Assume that agent 1 gets signal $S_1 = h$ and that agent 2 will report $\hat{s}_2 = S_2$ truthfully. Calculate the expected scores for agent 1 from reporting truthfully and from lying.

(c) **[5 Points]** Show how the 2 agents could usefully collude. *Hint: Construct an example, name the signal each agent receives and the signal each agent reports. Then show that the expected payoff for each agent is higher from colluding compared to not colluding.*

4. **[17 Points, MSc + 15 Points]** Reputation systems

   (a) **[12 Points]** Consider the repeated group prisoner's Dilemma game (see Chapter 13.3 of the class notes) with whitewashing. From the perspective of player 1 we have:

   - Players participate in a repeated n-player prisoner's Dilemma, i.e. each round they are randomly matched in pairs and their payoff is given by Figure 13.7.
   - All players start with a good reputation.
   - If a player defects, this player looses the good reputation.
   - At the beginning of each round, players without a good reputation have the option to pay an initiation fee f to whitewash, i.e. get a good reputation again.
   - All other players (except player 1) play the following strategy: Play C against an agent with a good reputation, play D otherwise.

   Suppose agents' discount their utility at a factor $\delta \in (0,1)$. Derive how high the initiation fee f has to be to deter player 1 from whitewashing.

   (b) **[MSc 15 Points]** Consider the repeated group prisoner's Dilemma with whitewashing. Verify that *pay-your-dues* (PYD) with a PYD-period of 1 is a Nash equilibrium of this game. *(Hint: Consider the 3 strategic options (I) to pay your dues, then cooperate forever, (II) defect forever, and (III) pay your dues, then defect once, then pay again, then defect again, etc. Then derive a lower bound on the discount factor δ that ensures that (I) has the highest payoff. Also argue why it is sufficient to consider only strategies (I), (II) and (III).)*

   (c) **[5 Points]** Use Figures 13.11 and 13.12 of the class notes to explain in what sense the introduction of DSR has improved informativeness of Ebay's reputation system. Also, which aspects remain problematic?