Lecture #12: Social Choice

Prof. Dr. Sven Seuken

23.4.2012

Housekeeping

- Questions? Concerns?
- Homework assignments
  - Mturk assignment: Mike found the solution!
    → need to create a “sandbox requestor account”
  - VISA information for Mturk assignment:
- Course evaluation: before the break!

Recap: Human Computation

Definition 11.1 (Human Computation). *Given a computational problem from a requester, design a solution using both automated computers and human computers (Law and von Ahn, 2011).*

- **Crowdsourcing** is a type of outsourcing tasks that are traditionally performed by an employee or contractor, to an undefined, large group of people or community (a crowd) through an open call.

- Crowdsourcing markets (e.g., Mturk, Crowdflower,…) are a great playing field to experiment with:
  - Different work allocation mechanisms
  - Different payment mechanisms
  - Different voting rules/social choice mechanisms

Duolingo

- http://duolingo.com

Today: Social Choice

The Set-up

- A set of alternatives $A = \{a, b, c, \dots\}$
- A set of agents $N = \{1,2, \dots n\}$
- Each agent i has a strict preference ordering $P_i \in L(A)$ over alternatives; for example:

$$a \succ_1 b \succ_1 c$$

$$b \succ_2 c \succ_2 a$$

$$c \succ_3 b \succ_3 a$$

- We let $P = (P_1, P_2, \dots, P_n) \in L(A)^n$ denote the preference profile (the joint preferences)
- **Social choice rule**: select best alternative $f(P) \in A$
- **Social ranking rule**: find best preference order $r(P) \in L(A)$

Mechanism Design vs. Social Choice

Theorem 12.2 (Gibbard-Satterthwaite). *If $|A| \geq 3$, any social choice rule that is onto A and strategyproof is dictatorial.*

- We have seen:
  - With |A|=2, majority rule is strategyproof.
  - With |A|>=3, Plurality can be manipulated
  - With |A|>=3, Borda can be manipulated
  - In fact: all interesting voting rules can be manipulated
- But: we still have to make a decision, even when |A| >= 3. → Social Choice Theory

The two Different Perspectives

- The subjective viewpoint:
  - With perfect information → still disagreement
  - Difference in votes due to different preferences

- The objective viewpoint:
  - With perfect information → full agreement
  - Difference in votes due to noise

- Examples…

Properties of Voting Rules

- Different voting rules still have different properties!
- Which ones do we want?
- We liked the majority rule for |A|=2, because…?

…the majority of voters will agree that the winner is indeed the best!

- Generalize to $|A| \geq 3$ …the alternative that beats every alternative by simple majority

Condorcet Consistency

- Condorcet Paradox:

$$a \succ_1 b \succ_1 c$$
$$b \succ_2 c \succ_2 a$$
$$c \succ_3' a \succ_3' b,$$

- **Condorcet Consistency**: a social choice rule is *Condorcet consistent* if it selects as the best alternative an alternative that beats (or ties) all others in pairwise comparisons, *when one exists*.

Some Popular Voting Rules

Definition 12.3 (Plurality). *Given $m \geq 3$ alternatives, each agent votes for its most preferred alternative. The alternative with the most votes is selected, breaking ties at random.*

Definition 12.4 (Plurality-with-run-off). *Given $m \geq 3$ alternatives, each agent votes for its most preferred alternative. Choose the two alternatives with the most votes (breaking ties at random), and run a second round of majority voting on these top two.*

Definition 12.5 (Plurality-with-elimination). *Given $m \geq 3$ alternatives, each agent votes for its most preferred alternative. Eliminate the alternative with the fewest votes (breaking ties at random). Each agent who cast a vote for the eliminated alternative casts a new vote. Repeat until only one alternative remains.*

Definition 12.6 (Borda). *Each agent submits a preference order on the m alternatives. Based on this, a score of $m-1$ is assigned to the top alternative, $m-2$ to the second, $\ldots$, down to 0 points to the last alternative. The alternative selected is the one with maximum total score, summing up across all agents (and breaking ties at random).*

Positional Scoring Rules

Definition 12.7 (positional scoring rule)**.** *A positional scoring rule gives a score of* $s_1, \ldots, s_m$ *to the alternative in position* $1, \ldots, m$ *of an agent's preference order, with* $s_j \geq 0$ *and* $s_1 \geq s_2 \geq \ldots \geq s_m$ *and* $s_1 > s_m$. *The selected alternative by the rule is the one with the maximum total score.*

- **Theorem:** No positional scoring rule is Condorcet consistent on three or more alternatives.

- But:
  - Positional scoring rules are simple
  - Widely used in practice

Axiomatic Approach

Definition 12.8 (unanimity). *A social ranking rule r satisfies unanimity if, given preference profile P, whenever all agents i agree that $a \succ_i b$, then $a \succ_r b$ in the social preference order $P_r = r(P)$.*

Definition 12.9 (independence of irrelevant alternatives (IIA)). *A social ranking rule r is independent of irrelevant alternatives if for any $a, b \in A$ and any two preference profiles P, P' that leave the preference between a and b unchanged for all agents (but can change other preferences), then the social preference order between a and b is unchanged.*

Theorem 12.3 (Arrow's impossibility theorem). *If $|A| \geq 3$, any social ranking rule r that satisfies unanimity and IIA is dictatorial.*

- Says nothing about strategy-proofness!!

→ Need to relax IIA → local consistency

Relaxing IIA

- Interval: some set of alternatives $A' \subset A$ form an interval given preference profile P and social ranking rule r, if alternatives A' are adjacent in the social rank order r(P).
- Example: Apply the Borda rule

$$3 \quad @ \quad a \succ b \succ c \succ d$$

$$2 \quad @ \quad b \succ c \succ d \succ a$$

$$2 \quad @ \quad c \succ d \succ a \succ b$$

- The social ranking is: $c \succ_r b \succ_r a \succ_r d$
- Now, drop alternative d:

Local Consistency

Definition 12.10 (local consistency). *A social ranking rule r satisfies local consistency if, for any preference profile P leading to social preference order $\succ_r$, applying r to a domain restricted to any set of alternatives A' on an interval in $\succ_r$ leaves the social preference order on A' unchanged.*

- Weaker than IIA
- Borda fails even local consistency
- Are there voting rules that are Condorcet consistent and satisfy local consistency?

→ Idea: select as the winning alternative one that is as close as possible to being a Condorcet winner!

Swap Distance

- The swap distance between P_i and P_i' is the minimum number of swaps of the position of adjacent alternatives to obtain P_i' from P_i.

- Example:

Dogson Rule (1/2)

Definition 12.11 (Dodgson rule). *Each agent submits a preference order on the m alternatives, providing preference profile P. Select as the winner an alternative that can be made a Condorcet winner in a preference profile P' that minimizes the swap distance from P.*

- Example:

$$a \succ_1 e \succ_1 d \succ_1 c \succ_1 b$$
$$e \succ_2 d \succ_2 c \succ_2 b \succ_2 a$$
$$b \succ_3 a \succ_3 c \succ_3 e \succ_3 d$$
$$b \succ_4 e \succ_4 d \succ_4 a \succ_4 c$$
$$c \succ_5 d \succ_5 a \succ_5 b \succ_5 e$$

- Alternatives a, b or e all have a Dogson score of 2
- To make a Condorcet winner:
  - → Change agents 2 and 4

Dogson Rule (2/2)

- **Theorem**: The Dogson rule is Condorcet consistent!

- **Proof**: ?

The Kemeny Rule

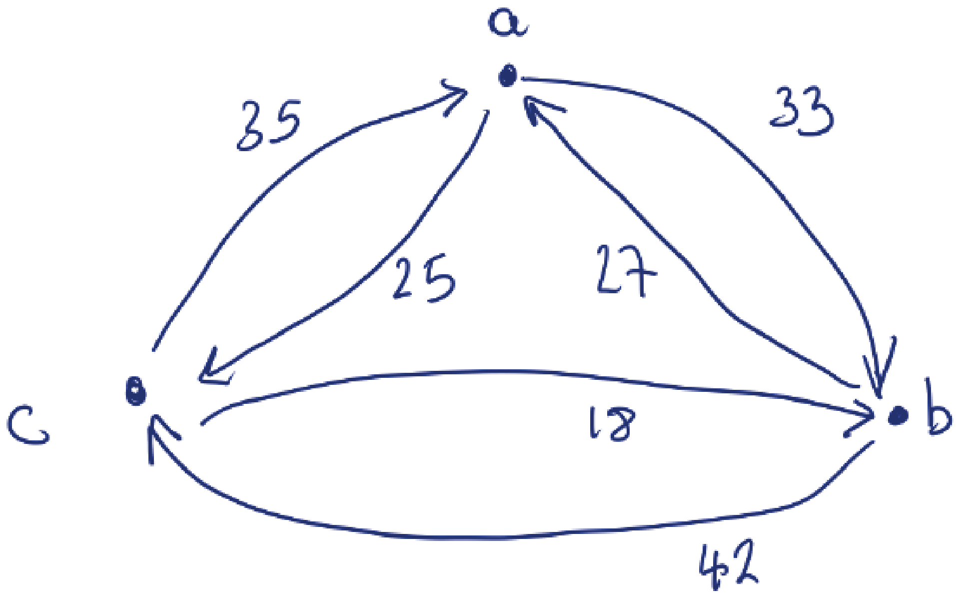
Definition 12.12 (Kemeny rule). *Each agent submits a preference order on the m alternatives, providing a preference profile P. Select as the social preference order P_r a preference order such that a profile that consists of P_r for every agent minimizes the swap distance from P. Select as the best alternative the best alternative in P_r.*

- Swap distance (P, P') = number of pairwise disagreements (P, P')
- Number of disagreements on (x,y) = number of agents – number of agreements on (x,y)

→ Kemeny: maximize total number of pairwise agreements

Kemeny Rule: Example

23	@	$a \succ b \succ c$
17	@	$b \succ c \succ a$
2	@	$b \succ a \succ c$
10	@	$c \succ a \succ b$
8	@	$c \succ b \succ a$



- No Condorcet winner: a beats b, b beat c, c beats a
- Compute total pairwise support>

$a \succ b \succ c$	$a \succ c \succ b$	$b \succ a \succ c$	$b \succ c \succ a$	$c \succ a \succ b$	$c \succ b \succ a$
100	76	94	104	86	80

Kemeny Rule (ct.)

- **Theorem**: The Kemeny rule is Condorcet consistent!

- Proof:

More Properties of the Kemeny Rule

- **Neutrality**: the outcome is invariant to the identity of the alternatives.
- **Anonymity**: the outcome is invariant to the identity of the agents.

Definition 12.13 (reinforcement)**.** *A social ranking rule r satisfies reinforcement if when two pools of agents with preference profile P and P' respectively generate the same social preference order $P_r = r(P) = r(P')$ then then social preference order is the same after the pools are merged, with $P_r = r(P \cup P')$.*

- **Theorem:** The Kemeny rule is the unique social ranking rule that satisfies anonymity, neutrality, unanimity, reinforcement and local-consistency.

The Statistical View

- Consider the “objective” viewpoint:

Any difference in votes is due to noise

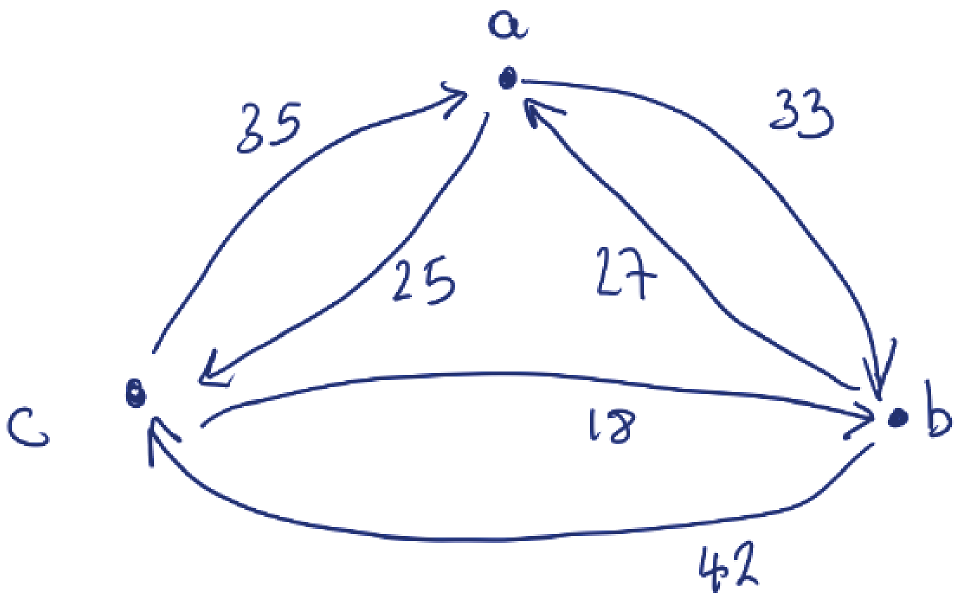
- The Kemeny rule identifies the alternative that is “most likely” to be the best alternative!

→ Strong statistical support. This is good, in particular when there is no strategic voting!!

Computational Considerations

- Winner determination = which alternative (or which ranking) gets selected?
- Many voting rules are easy to compute
  - Majority
  - Plurality
  - Borda
- But Kemeny is more difficult…

Computing Kemeny



- For every pair of alternatives, select one edge, such that the path is acyclic
- Maximize the total weight of the selected edges

$$\max_{x_{jk}: j \neq k} \sum_{j,k: j \neq k} w_{jk} x_{jk}$$

$$\begin{aligned} \text{s.t.} \quad & x_{jk} + x_{kj} = 1, && \text{for all } j, k, \ j \neq k \\ & x_{jk} + x_{kl} + x_{lj} \leq 2, && \text{for all } j, k, l, \text{ and } j \neq k \neq l \\ & x_{jk} \in \{0, 1\}, && \text{for all } j, k, \ j \neq k \end{aligned}$$

Complexity of Kemeny Rule

- Can be formalized as IP → can compute in worst-case exponential time
- It is known that computing Kemeny rankings is NP-hard → cannot do better than worst-case exponential time
- In practice: 25 alternatives in 5 seconds
- Some very hard instances with 40 alternatives

The Schulze Rule

- Alternative to Kemeny
- Can be computed in polynomial time O(m^3).
- Also Condorcet consistent.

Course Evaluation

University of Zurich[UZH]

Academic Program Development

Your course – your opinion

Online questionnaire for Bachelor and Master's students in this course

from 23 April – 6 May 2012

You will receive an e-mail with further information about the survey when it begins on 23 April 2012.

Thank you for participating!

Contact information: Sarah Colombo, lvb@lehre.uzh.ch