Computation and Economics - Spring 2012
Homework Assignment #06: Mechanism Design and Combinatorial Auctions

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Out Thursday, April 5, 2012
Due **12:15** sharp: **Thursday, April 19, 2012** (in class)
We strongly encourage typed submissions.

[**BSc 100 Points, MSc 120 Points**] This is a single-person assignment. Points will be awarded for clarity, correctness and completeness of the answers. Reasoning must be provided with every answer. You are free to discuss the problem set with other students. However, you should not share answers. **Copying will be penalized**.

1. [**11 Points**] Mechanism Design: VCG Mechanism

   Santa Claus, Rudolf and the Easter Bunny meet to discuss cost saving measures. They find out that Christmas and Easter can be merged on one of the dates with neglegible additional effort, e.g. if both festivities are held on December 24 simultaneously, the effort to Santa Claus and Rudolf does not change, but the Easter Bunny has more leasure time and vice versa. To decide which date to pick, they use a VCG mechanism.

   (a) [**4 Points**] Consider the following values when both festivities are combined on the respective occasion:

	Christmas	Easter
Santa Claus	0	200
Rudolf	0	100
Easter Bunny	250	0

   Describe the outcome of the VCG mechanism in terms of the reports $\hat{v}$, the allocation $x(\hat{v})$ and the transfers $t_i(\hat{v}), i = 1, 2, 3$.

   (b) [**5 Points**] If Santa Claus and Rudolf collude, what could they do to be jointly better off?

   (c) [**2 Points**] Does the manipulation described in the previous question violate the DSIC property of the VCG mechanism? Why or why not?

2. [**18 Points**] Mechanism Design: VCG Mechanism II

   (a) [**10 Points**] Construct an example where VCG is not individually rational. *Hint: Construct an example that violates condition P1 or P2 of Theorem 8.2 and show that agents may have negative utility.*

(b) **[4 Points]** Consider the *public project* setting from Example 8.5. Construct a set of values v_i for agents, such that the mechanism will run a deficit.

(c) **[4 Points]** Show that this is not a contradiction to the no-deficit property of VCG from Theorem 8.3.

3. **[12 Points]** Mechanism Design: Revelation Principle

The *Dutch Auction* is defined in Definition 6.10.

(a) **[1 Points]** Refresher - Comment on the following points in 2-3 sentences each:

- The Dutch Auction is not a direct-revelation mechanism.
- The Dutch Auction is strategically equivalent to a first-price, sealed-bid auction.
- What is a BNE of the Dutch Auction if agents' values are IID and uniformly distributed?

(b) **[8 Points]** Let agents' private values be IID and uniformly distributed. Construct a truthful direct revelation mechanism that implements the social choice function of the Dutch Auction in a BNE. Describe all elements of Figure 8.4 of the class notes.

(c) **[3 Points]** If communication from agents to the mechanism is costly, what is the cost of the direct mechanism (in terms of number total number of messages sent from agents to the mechanism)? What about the cost of the indirect mechanism?

4. **[14 Points, MSc +4 Points]** Mechanism Design: Impossibility Results

Consider the election of a political representative out of m candidates (alternatives) by n voters (agents).

For 3 candidates, a simple form of the *Borda scoring rule* works as follows: The candidate gets 0 points for every agent that votes him last, 1 point for every agent that votes him second and 2 points for every agent that votes him in first position. The candidate with the most points wins.

The numbers $0, 1, 2$ can be interpreted as weights. In the following formalization of the Borda rule, they may also depend on the voter.

- Collect preference orderings from all agents
- Define the weight of being ranked in k^{th} position by agent i as $w_{i,k} := m - k$. (Note that the weight may depend on the agent i, but in this definition it does not.)
- Define an indicator that is 1 if candidate a holds k^{th} position in the ranking of agent i, i.e.
$$x_i(a,k) := \begin{cases} 1, & \text{if } a \text{ is } i\text{'s } k^{th}\text{choice,} \\ 0, & \text{else.} \end{cases}$$
- For each candidate a compute the sum of weights
$$X(a) = \sum_{i=1}^{n} \left(\sum_{k=1}^{m} w_{i,k} x_i(a,k) \right).$$
For the weights defined above, this means: for each time the candidate is ranked first, he receives $m-1$ points, $m-2$ for each second ranking, and so on

- Select the candidate a with the highest score $X(a)$, break ties at random

(a) **[7 Points]** If all preference orderings are allowed, is this mechanism truthful with the weights defined as above? Why or why not?

(b) **[7 Points]** How can we change the weights $w_{i,k}$ to yield a truthful mechanism that is onto? *Hint: To get onto, ensure that the outcome actually depends on the agents' reports and that any outcome can be attained.*

(c) **[MSc 4 Points]** Argue that this mechanism is Pareto optimal.

5. **[MSc 16 Points]** Mechanism Design: Single Peaked Preferences

Consider the median selection mechanism. Let n denote the number of agents. Suppose, in addition, that the mechanism designer can position $n-1$ "phantom peaks" before the peaks from the agents are received.

(a) **[MSc 8 Points]** Explain why the median selection mechanism on the resulting $2n-1$ peaks remains strategyproof?

(b) **[MSc 8 Points]** Suppose we would like the mechanism to always select the peak that is at the k^{th} position from the right. Describe a method to position the phantom peaks to implement this outcome.

6. **[25 Points]** Combinatorial Auctions: Bidding Languages

Consider the following setting: Mobile phone companies can bid for 2 types of spectrum rights (license I, license II) in each of 2 cities (Duckburg, Springfield); hence there are 4 licenses in total: (I,D.), (II,D.), (I,S.), (II,S.).

(a) **[15 Points]** If two companies DagoTalks and BurnSMS use the XOR language to submit their bids, formulate the auctioneers allocation problem as an integer program. Your formulation should be general, i.e. use variables to encode valuations and preferences.

(b) **[5 Points]** If the business case for company BurnSMS is as follows:

- To service a city, both licenses in that city are needed due to the inferior technology.
- Servicing Springfield will yield profits of 100.
- Servicing Duckburg generates profit of 50 only if Springfield is not serviced.

Give an OR* representation of the truthful bid by BurnSMS. Explain your reasoning.

(c) **[5 Points]** If the business case for company DuckTalks is as follows:

- At least one local license is needed to service a city.
- Servicing a city with one license yields a profit of 100.
- With two licenses for the same city the profit is slightly higher at 105, because coverage improves slightly.
- Offering service in both cities yields commuters as additional customers. In this case the profit is twice the sum of the profits from each of the cities.

Which bidding languages can be used to submit a bid? Which are inadequate? Provide reasoning.

7. **[20 Points]** Combinatorial Auctions: VCG, Greedy, Critical Payment

Consider an instance of the single-minded CA on four goods $\{A, B, C, D\}$, and with three bidders, with valuations

Bidder	(Bundle, Valuation)
1:	(A,10)
2:	(ABCD,19)
3:	(B,8)

(a) **[10 Points]** What is the outcome of the mechanism that uses the greedy winner determination algorithm (see Definition 10.3) and determines VCG prices? Show all work and give two useful manipulations.

(b) **[10 Points]** What is the outcome of the "greedy + critical payment" mechanism defined in Definition 10.4? Show all work. Show that the two manipulations are no longer useful.