Computation and Economics - Spring 2012
Homework Assignment #10: Prediction Markets, Transitive Trust Systems, Electronic Currencies

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Out Thursday, May 24, 2012
Due **12:15** sharp: **Thursday, May 31, 2012** (in class)
We strongly encourage typed submissions.

[**BSc 100 Points, MSc 120 Points**] This is a single-person assignment. Points will be awarded for clarity, correctness and completeness of the answers. Reasoning must be provided with every answer. You are free to discuss the problem set with other students. However, you should not share answers. **Copying will be penalized**.

1. [**22 Points**] Prediction Markets - Logarithmic Market Scoring Rule

   Consider a prediction market with 2 outcomes ω_1 and ω_2. Agents can purchase contracts S_1 and S_2, each worth \$1 if the event ω_1 or ω_2 is observed, respectively. The contracts are priced using the Logarithmic Market Scoring Rule as in Example 15.1 of the Notes. Suppose, $\beta = 5$, the initial position is $x = (0, 0)$ and initial prices are at $p = (0.5, 0.5)$.

   (a) [**6 Points**] If agent 1 buys 15 shares of S_1, how much will he pay? What will be the new prices?

   (b) [**6 Points**] After the first trade is executed, suppose, agent 2 believes that the probability of ω_1 occuring is 0.8 and has unlimited funds. Which and how many contracts would he have to buy, so that the market reflects his beliefs? How much would this cost him?

   (c) [**10 Points**] If after both trades are executed, the event ω_1 is observed, what is the profit/loss to each agent and to the market maker?

2. [**MSc 15 Points**] Prediction Markets - Bounded Loss

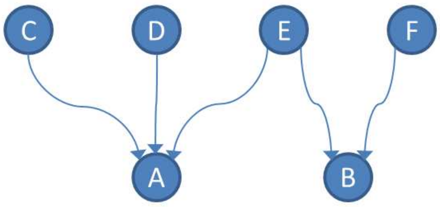
   So far we have considered the LMSR market maker. Now, consider a prediction market with n possible outcomes $\Omega = \{\omega_1, \ldots, \omega_n\}$ and prices determined using a Quadratic Market Scoring Rule with factor $\beta > 0$, i.e. for any $\omega \in \Omega$ we have

   $$R(\omega, \Pr) = 2\beta \Pr(\omega) - \beta \sum_{k=1}^{n} \Pr(\omega_k).$$

   Derive a lower bound for the loss of the market maker. *(Hint: The worst case for the market maker occurs when the agents trade such that the probability of an event ω_0 is set to 1 (and 0 for all other events) and the event ω_0 actually occurs.)*

3. [**20 Points**] Transitive Trust Mechansims - Hub-and-Authority

Consider the following web graph.



(a) [**10 Points**] Compute the values after two rounds of the hub-and-authority algorithm (in each round, the authority and then the hub scores are updated, see Definition 17.1 of the Notes.) Show the values before and after the normalization step, which occurs after the end of the second round (and makes the total hub score, and the total authority score, each sum to 1.)

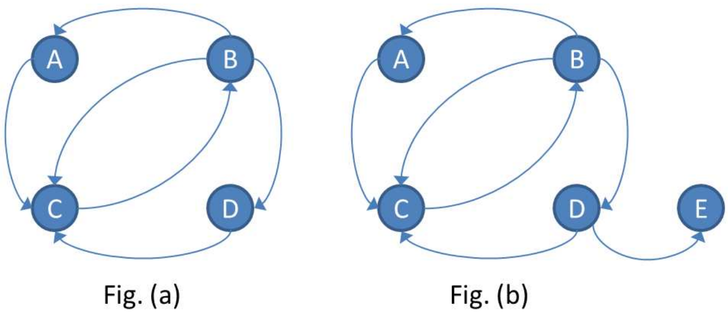
(b) [**10 Points**] Now consider strategic behavior where you create a new webpage X and also add a node Y that will be used to vote for X and give it some authority. Compute the normalized scores after two rounds of hub-and-authority for each of the following two options:

- Add X and Y and have Y only point to X
- Add X and Y and have Y point to A, B and X

Which is the best for X? Provide some intuition for why this is better.

4. [**21 Points**] Transitive Trust Mechansims - Page Rank

Consider the web graphs given by figures (a) and (b).



(a) [**6 Points**] For Figure (a), show that values $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$ (for A, B, C, D, respectively) are the equilibrium PageRank values for the basic PageRank mechanism (i.e., these are the values reached in the limit).

(b) [**5 Points**] What are the limit values for the basic page rank mechanism used on figure (b)? What drawback of the page rank mechanism becomes apparent in this example?

(c) [**10 Points**] Determine the approximate limit values for the scaled page rank mechanism with scaling factor $s = 0.5$ used on figure (b) and explain how the drawback is resolved. *(Hint: It is sufficient to simulate 30 rounds of the mechanism, e.g. in Excel. Provide proof of your calculation or simulation.)*

5. [**15 Points, MSc +5 Points**] Virtual Currencies - iOwe

   (a) [**10 Points**] Does iOwe use transitive trust? Explain why or why not?

   (b) [**5 Points, MSc +5 Points**] Consider the policy *P2: Chaining of Trust* as described on page 8 of the Notes on Electronic Currencies. Suppose, an iota is transferred along the following chain of agents

$$A \to B \to C \to D \to E \to A,$$

   where E redeems the iota with A in the last step.

      i. [**5 Points**] How many bilateral barter exchanges are needed to build the necessary trust for this chain to work? Explain.

      ii. [**MSc 5 Points**] In general, how many bilateral barter exchanges are needed for a chain involving n different agents to work? Derive a general formula!

6. [**22 Points**] Virtual Currencies - BitCoin

   (a) [**5 Points**] Consider figure 18.2 in the Notes. When a BitCoin is transferred from agent 1 to agent 2, explain why the public key of agent 2 is included in the transaction.

   (b) [**9 Points**] Imagine an attacker A has a share of $p \in [0, 1]$ of the total computing power in the system, i.e. 0 for none, 1 for all. Further, assume that the probability of finding the next acceptable block is equal to p. Determine the probability that a double-spending-attack by A is successful and plot this probability as a function of p (put $p \in [0, 1]$ on the x-axis, and the probability of a successful attack on the y-axis).

   (c) [**8 Points**] Name two drawbacks of iOwe and explain how BitCoin fixes these problems.