Lecture #2: Introduction to Game Theory

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27.2.2012

Outline

1. Recap of last lecture
2. Go over Game Theory concepts
3. Play some in-class experiments
4. Discussion
5. Questions

Quick Recap

- Braess' Paradox: **Incentives** matter!
  → rational behavior leads to socially sub-optimal outcome (in **equilibrium**)
- PDP-1 Auction:
  - Allows users to express their preferences
  - Use a "price signal" to coordinate users
  - Leads to "efficient" use of the common resource: → **No** tragedy of the commons
- 1st –price vs. 2nd-price Auctions:
  - Strategic bidding leads to numerous problems
  - Want: incentive compatibility
  - Ideally: dominant strategies
- In this course: take a design perspective

- Econ+CS = incentive constraints + computational constraints

Game Theory Concepts

- PD Game
  - Payoffs
  - Utilities and affine transformations
- Pareto-optimality:
  - Definition
  - what should the prisoner do? (Q1)
- Where do we have PD games in real life?

- Dominant Strategies
- Iterated elimination of dominated strategies
- Nash equilibrium
  - Pure strategies
  - Mixed strategies (how do I see one is a NE)
  - Matching Pennies: why no pure-Nash? (Q2)
  - Multiplicity
- Zero-sum games:
  - no pure Nash!
  - Maximin Theorem!
  - Zero-sum vs. constant-sum

- Graphical games
  - Why graphical?
  - Game with 3 players in normal-form?
  - Example: 4 players with 4 actions: NF vs. GGs

The Prisoner's Dilemma Game

		Player 2: C	Player 2: D
Player 1	C	3, 3	0, 5
	D	5, 0	1, 1

1. Story?
2. Positive affine transformations of utility functions…

Dominant Strategies & Pareto Optimality

- Dominant Strategies
- Pareto-optimality

→ What should the prisoner do in the PD game?

		Player 2: C	Player 2: D
Player 1	C	3, 3	0, 5
	D	5, 0	1, 1

Let's play it…

Player 1 \ Player 2	C	D
C	3, 3	0, 5
D	5, 0	1, 1

Let’s play it: The Sharing Game

- You have $5.
- Every dollar you donate will be matched with $1.
- The total contribution will be divided equally across the class.
- How much will you donate?
- The idea is to maximize payoff.

- Enter a donation between 0 and 5.

Iterated Elimination of Dominated Strategies

		Player 2		
		L	M	R
	U	4, 3	5, 1	6, 2
Player 1	M	2, 1	8, 4	3, 6
	D	3, 0	9, 6	2, 8

Nash Equilibrium

- Pure strategies
- Mixed strategies (how do I see one is a NE)
- Matching Pennies: why no pure-Nash? (Q2)
- Multiplicity

Pure-Strategy NE

Definition 2.6 (Nash equilibrium). *Action profile $a^* = (a_1^*, \ldots, a_n^*)$ is a pure-strategy Nash equilibrium (NE) of normal-form game (N, A, u) if, for all i,*

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*), \quad \text{for all } a_i \in A_i. \tag{2.2}$$

The Matching Pennies Game

		Player 2	
		H	T
Player 1	H	1, −1	−1, 1
	T	−1, 1	1, −1

Mixed-Strategy NE

Definition 2.8 (mixed-strategy Nash equilibrium). *Mixed strategy profile* $s^* = (s_1^*, \ldots, s_n^*)$ *is a Nash equilibrium in game* (N, A, u) *if, for all* i,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \quad \textit{for all } s_i \in \triangle(A_i). \tag{2.4}$$

The Game of Chicken

- Multiple equilibria…
  - (Y,S)
  - (S,Y)
  - ((Y=2/3, S=1/3),(Y=2/3, S=1/3))

		Player 2	
		Y	S
Player 1	Y	0, 0	0, 2
	S	2, 0	−4, −4

The Luggage Game

- Two travelers have identical luggage. Their luggage is damaged by an airline and the airline offers to recompense them for their luggage.
- The travelers may ask for any dollar amount (m_1, m_2)between \$2 and \$100. There is one catch:
  - If they ask for the same amount, then that is what they will both receive.
  - If they ask for different amounts , then whoever asks for the lower amount m_1 will get m_1 + \$2, and whoever asked for the higher amount will get m_1 - \$2.

Playing the Luggage Game

- Give a number between 2 and 100!
- You are matched with each other player in the class, and your average score is computed.
- Results…?
- Interpretation?

Playing the Luggage Game - Again

- Can explain that 100 is weakly dominated by 99
  → iterate this thinking gets to 2,2 as equilibrium.

- 2005 Game Theory society played the game. Of the 45 who played, 33 played >= 95, 38 >= 90, 3 played 2.
- Best payoff was 97 (85.1), worst was 2 (3.92).

- Let's play again!
- Now play it again with:
  – $m_1 < m_2$: pay $m_1 + 25$ and m_1-25
  – $m_1 < m_2$: pay $m_1 + 100$ and m_1-100

Problems of Game Theory

- Players may not act rationally.
  - It’s proven that they don’t?
  - Emotions, feelings, social preferences?
  - What do we mean by “act rationally”?
  - How else do agents act?
- Common knowledge assumption
- Complex problems cannot be solved by a human mind
- Determine the utility of players?
- Agents must behave according to a specific utility function?
- The “games” in real life are often not formally specified
- Modeling complexity
- Calculation complexity
- A rational strategy may not be good for a game against irrational players when played just once

Individual Rationality as a Useful Approximation

Roth, A.E. "Comments on Tversky's 'Rational Theory and Constructive Choice'," **The Rational Foundations of Economic Behavior**, K. Arrow, E. Colombatto, M Perlman, and C. Schmidt, editors, Macmillan, 1996, 198-202.

- Rational models as useful approximations
  - All models are approximations, and false at some level of detail
  - But they serve as good approximations for human behavior → good predictions

- Different models:
  1. Risk-neutral economic man
  2. Expected utility maximizing man
  3. Almost rational man
  4. Psychological man
  5. Neuro-biological man

- The potential for non-rational models in economics
  - Extensions to utility theory, plug-compatible → predictions? Complexity?
  - Adaptation, learning → predictions?
  - How important are the “non-rational” actions observed in experiments?
  - The future will show…!

Hyper-Rationality

- Hal Varian (Economics Professor and Google's Chief Economist) in 1995:

  "… hyper-rationality may actually be [an] appropriate model for software agents…The whole framework of game theory and mechanism design may well find its most exciting and practical application with computerized agents rather than human agents."