

Exercise 1: Formal Grammars and Languages

Formal Methods II, Fall Semester 2013

Solution Sheet

Formal Languages

1. (6 points)

(a) This *regular* language can be generated by the grammar

$$\begin{aligned} S &\rightarrow aS \mid aT \\ T &\rightarrow bT \mid \epsilon \end{aligned}$$

(b) This *regular* language can be generated by the grammar

$$\begin{aligned} S &\rightarrow aT \mid bS \mid cS \mid \epsilon \\ T &\rightarrow aS \mid bT \mid cT \end{aligned}$$

(c) This *context-free* language can be generated by the grammar

$$S \rightarrow 0S1 \mid 1S0 \mid \epsilon$$

(d) (2 bonus points) This *context-free* language can be generated by the grammar

$$S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon$$

Regular Expressions

2. (3 points) The only possible solutions are:

- (a) 011 or 001
- (b) 1001
- (c) 0101 or 1010

3. (6 points) Some possible solutions include:

(a) $0^*1(0 | 1)^*00$ or $1(0 | 1)^*00$ if you exclude initial 0's.

Note that $(0 | 1)^*00$ contains 00, which is strictly speaking not positive.

(b) $(1 | 01^*0)^*$

(c) $(0 | 100)^*$

(d) $\underbrace{(b|abb)^*}_{prefix} \underbrace{(aa(bb + |a) * b^?)}_{aa \wedge \neg aba} \underbrace{|aba(b + a?)^*)}_{aba \wedge \neg aa}$

(e) b^*a^*

(f) $(b | ab)^*a^?$

4. (3 points)

(a)

$$\begin{aligned} S &\rightarrow aT \\ T &\rightarrow bT | aS | a \end{aligned}$$

(b)

$$\begin{aligned} S &\rightarrow aS | T \\ T &\rightarrow bT | U \\ U &\rightarrow aU | \epsilon \end{aligned}$$

(c) The shortest grammar is in fact *left* regular:

$$S \rightarrow S0 | S1 | 01$$

5. (2 points)

(a) `grep -E "^[[:alpha:]][[:alnum:]]*" *`

(b) `grep -E "^[[:alpha:]][[:alnum:]]{4,}" *`

(c) (1 bonus point)

`grep -E "(^|[:space:])[[:alpha:]][[:alnum:]]{4,}($|[:space:])" *`