Computation and Economics - Fall 2014
Assignment #3: Mechanism Design

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Out Tuesday, October 14, 2014
Due **12:15** sharp: **Tuesday, October 21, 2014**
Submissions should be made in writing before the beginning of the lecture.

[**Total: 100 Points**] This is a single-person assignment. Points will be awarded for clarity, correctness and completeness of the answers. Reasoning must be provided with every answer, i.e., please show your work. You get most of the credit for showing the way in which you arrived at the solution, not for the final answer. You are free to discuss the assignment with other students. However, you are not allowed to share (even partial) answers with each other, and **copying will be penalized**.

1. [**23 Points**] VCG mechanism.

   Alice and Bob are thinking about asking their boss for a coffee machine for their office. Suppose that Alice appreciates coffee more than Bob, i.e., she would have value $40 for having the machine, and Bob's value is $20. Purchasing the machine would result in a negative value for the Company of $-$50. If no machine is set up, all have value $0.

   Suppose a VGC mechanism is used to determine whether or not to set up the coffee machine and how much the agents A(lice), B(ob), and C(ompany) have to pay.

   (a) [**7 Points**] If all agents follow their dominant strategy, what is the outcome of the mechanism in terms of the reports $\hat{v}$, the allocation $x(\hat{v})$, and the transfers $t_A(\hat{v})$, $t_B(\hat{v})$, and $t_C(\hat{v})$?

   (b) [**9 Points**] If Alice and Bob collude, how can they save money?

   (c) [**7 Points**] Even if Alice and Bob do not collude, why might this mechanism be problematic in practice?

2. [**41 Points**] Revelation Principle.

   A stray dog has been living in a dog shelter for some time now, when two guys show up to adopt it. Each of them can be either *good* or *evil*. A good guy cares for the dog a lot, i.e., he would prefer to adopt the dog himself, his second choice is that the other person adopts the dog, and his last choice would be for the dog to remain in the shelter. A bad guy also prefers to adopt the dog, but would rather see the dog left in the shelter than adopted by the other person. The employee of the dog shelter can decide to give the animal to one of the guys or keep it in the shelter (consider the employee as a mechanism designer, not as an agent).

(a) [**8 Points**] Give a formal representation of the problem, i.e., specify the set of agents N, a set of outcomes O, and a set utilities U for the agents.

(b) [**20 Points**] Suppose, no payments are possible. The employee wants to achieve the following result:

- give the dog to a good guy,
- but keep the dog if there are only evil guys.

Show that the employee cannot design a mechanism that implements this result in a dominant strategy equilibrium? *Hint: use the revelation principle.*

(c) [**13 Points**] In a biblical story, King Solomon is facing the following situation: he is approached by two women, who are fighting over a baby, both claiming to be its mother. The king announces that he will split the baby in half. The false mother does not protest this judgment, while the real mother pleads that the baby should be left in one piece and given to the other woman. The King can thus recognize the real mother and give the baby to her.

Explain how the situation of the dog shelter and the situation King Solomon are related. Why was the King successful in this situation?

3. [**36 Points**] VCG mechanism ... again.

Two flatmates are very busy with work and are therefore considering to engage a cook for their daily lunch. The cook offers to provide an early lunch at 12:30 or a late lunch at 14:00. The cook has to exert effort, i.e., he has values -16 for preparing early lunch and -17 for late lunch. Suppose that both flatmates have value 15 for the early lunch and value 10 for the late lunch. If the cook is not engaged, all agents have value 0.

(a) [**11 Points**] Assuming that agents follow their dominant strategy, describe the outcome of the VCG mechanism in terms of the reports $\hat{v}$, the allocation $x(\hat{v})$, and the transfers $t_i(\hat{v})$, $i \in \{fm1, fm2, cook\}$.

(b) [**14 Points**] A third flatmate ($fm3$) moves in, who has the same values for early and late lunches as the other flatmates. However, $fm3$ decides to attend the lecture on Economics and Computation at UZH (12:15-13:45 every day! ☺), and thus he can only take late lunches because of the lecture. Suppose the *cook* thinks it is really rude to cook for only some of the flatmates and therefore will only prepare a lunch if *all* flatmates are present.

Describe the outcome of the VCG mechanism in terms of the reports $\hat{v}$, the allocation $x(\hat{v})$, the transfers $t_i(\hat{v})$, $i \in \{fm1, fm2, fm3, cook\}$. What does $fm3$ think about this outcome? Explain why this is not a contradiction to Theorem 7.6.

(c) [**11 Points**] Suppose now that the semester is over, so $fm1, fm2, fm3$ can make it to all lunches. What is the outcome (i.e., allocation and payments) of VCG in this case? If the mechanism runs a deficit, why is this not a contradiction to Theorem 7.6?

4. [**Bonus Assignment**] Nash Implementation in Full Information Environments

Consider a full information environment, i.e., all n agents know their own utilities *and* the utilities of all other agents, but the mechanism center does not know them. Let O be the set

of outcomes and suppose that there is a particular outcome o_0 that all agents like the least, i.e., for all agents $i \in N$ we have $u_i(o_0) = 0$ and $u_i(o) > 0$ for all $o \in O\backslash\{o_0\}$.

(a) Show that there exists a social choice function that cannot be implemented in Nash equilibrium by a direct revelation mechanism where all agents report their own utility. *Hint: assume simple type and outcome spaces and construct a counterexample.*

(b) Show that any social choice function that selects outcomes from $O\backslash\{o_0\}$ can be implemented in Nash equilibrium in an indirect mechanism. *Hint: the mechanism can ask agents about the utilities of other agents as well. Hint 2: truthful reporting does not have to be a dominant strategy.*

(c) Give two reasons why such a mechanism may not be very useful in practice. *Hint: think about the informational assumptions and other Nash equilibria of the mechanism.*