Computation and Economics - Fall 2014
Assignment #2: Auction Theory

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Out Tuesday, October 7, 2014
Due **12:15** sharp: **Tuesday, October 14, 2014**
Submissions should be made in writing before the beginning of the lecture.

[**Total: 100 Points**] This is a single-person assignment. Points will be awarded for clarity, correctness and completeness of the answers. Reasoning must be provided with every answer, i.e., please show your work. You get most of the credit for showing the way in which you arrived at the solution, not for the final answer. You are free to discuss the assignment with other students. However, you are not allowed to share (even partial) answers with each other, and **copying will be penalised**.

1. [**47 Points**] Second Price Auction and Reserve Price.

   (a) [**27 Points**] Two bidders, say bidder 1 and bidder 2, with independent private values (IPV), compete for an item in a Second Price Auction. Each of them has either value \$10 or \$20 with equal probabilities. Assume that if both submit equal bids, bidder 1 wins the auction and pays exactly her value.

      i. [**7 Points**] What are the dominant strategies for our bidders? Do they change if a reserve price is introduced?

      ii. [**8 Points**] What is the expected revenue for the seller in case bidders bid according to their dominant strategies?

      iii. [**6 Points**] Assume bidders bid according to their dominant strategies. Find a reserve price $r > 0$ that the seller can use for increasing his expected revenue.

      iv. [**6 Points**] Suppose the seller adopts this reserve price to increase his revenue. Is there any downside to this? (Hint: think efficiency.)

   (b) [**20 Points**] Let's now consider a second price auction with only one bidder whose value w is a random variable with uniform probability distribution on interval $[\$0, \$20]$. Thus, without a reserve price, the revenue of this auction will always be 0.

      i. [**9 Points**] Assuming that our bidder bids truthfully, what is the reserve price $r > 0$ that maximises the expected revenue of the seller?

      ii. [**11 Points**] Suppose that the seller attributes a value u to his article, i.e., he will lose value u if the item is sold. What is the reserve price $r > 0$ that maximises his expected revenue?

2. **[31 Points]** First Price Auction.

Two bidders compete for an item in a First Price Auction. They have independent private values distributed according to a probability density function $g(w) = 2w$ on $[0, 1]$. (Note: consider these values as IPV values even though $g(0) = 0$, which is not consistent with Definition 6.4.)

(a) **[22 Points]** Verify that the strategy profile $s^* = (s_1^*, s_2^*)$ with

$$s_i^*(w_i) = \frac{2}{3} w_i \quad \forall i \in \{0, 1\}$$

is a Bayes-Nash equilibrium for the auction.

(b) **[9 Points]** Consider the same setting, but with three bidders, each with independent private values, distributed according to g. What is the expected revenue of the seller in this case?
(Hint: if the bidders have values w_1, w_2, w_3 independently distributed according to g, then $\mathbb{E}[\min(w_1, w_2, w_3)] = 16/35$, $\mathbb{E}[\text{median}(w_1, w_2, w_3)] = 24/35$, $\mathbb{E}[\max(w_1, w_2, w_3)] = 6/7$.)

3. **[22 Points]** *eBay*.

Suppose that there are two separate auctions on *eBay*: an auction for a *pair of skis* ends on Tuesday evening, and another auction for a *pair of ski poles* already ends Monday evening. Your value for the *pair of skis* alone is $v_{skis} = \$100.00$, for the poles alone you have value $v_{poles} = \$20.00$, but if you get both items, your value is $v_{skis+poles} = \$200.00$ for the package. Assume *eBay* would use a Second Price Auction

(a) **[7 Points]** Is it a dominant strategy to bid your true values for the individual items (v_{ski} and v_{poles}) in the two separate auctions? Explain.

(b) **[8 Points]** If there are several bidders with different valuations, will truthfully bidding the individual values by all bidders leads to an efficient outcome? Explain.

(c) **[7 Points]** How does *eBay* currently address this problem, if at all?

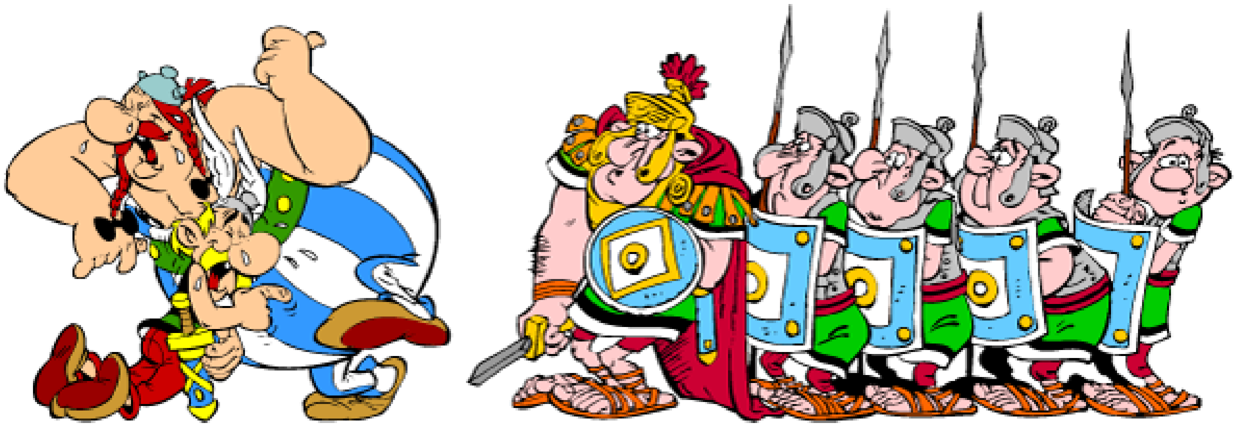

Figure 1: Preparation for the final battle

4. [**Bonus Assignment**] The year is 50 BC, and all Gaul is occupied. All of it? Only one small village of indomitable Gauls still holds out against the invaders. But how much longer can Asterix, Obelix and their friends resist the mighty Roman legions of Julius Caesar? Figure 1 depicts the situation just before the final battle ...

   Suppose both, the Romans and the Gauls, can invest resources: while the Romans spend their sestertii on fresh soldiers, Asterix and his friends need to collect valuable ingredients for the magic potion. We model this situation as an auction:

   - Winning yields a premium W to the winning party. The losing party gets nothing.
   - Each party i can secretly invest effort $b_i \in [0, \infty)$, which is the *bid* of that party.
   - During the final battle, Gaulish and Roman effort cancel each other out at a ratio of 1 to 1 until all effort of one side is exhausted.
   - The party that has effort left at the end wins the battle and can keep the premium W as well as the unused effort.

   (a) Formalize the auction game.

   (b) Show that there is no DSE in this auction.

   (c) There are 2 pure strategy Nash equilibria. Find them.