Computation and Economics - Fall 2014
Assignment #5(a): Linear and Integer Programming

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Out Tuesday, October 28, 2014
Due **12:15** sharp: **Tuesday, November 4, 2014**
For submission format, check description below.

[**Total: 40 Points**] This is a single-person assignment. Points will be awarded for clarity, correctness and completeness of the answers. Reasoning must be provided with every answer, i.e., please show your work. You get most of the credit for showing the way in which you arrived at the solution, not for the final answer. You are free to discuss the assignment with other students. However, you are not allowed to share (even partial) answers and source code with each other, and **copying will be penalized**.
Every student has to submit his/her write up before the beginning of the lecture. Before that deadline, the .java-files should be sent via email to `brero@ifi.uzh.ch` in a zip-file.

1 Setup

In this assignment, you will implement different linear and integer programs using Java/JOpt. To set everything up, do the following steps (assuming you will use Eclipse):

1. download CPLEX binaries from the link you can find on 'Installation guide CPLEX.pdf' (check on OLAT);
2. install CPLEX following this installation guide;
3. find the library cplex.jar in the installed folder of cplex - path: `IBM/ILOG/CPLEX_Studio126/cplex/lib/`;
4. find the library jopt.jar in the folder jopt (check on OLAT) - path: `jopt/lib/`;
5. import both libraries into your Eclipse project;
6. find the documentation in the folder jopt - path: `jopt/doc/javadoc/`;
7. find examples of integer programs in the folder Examples (check on OLAT);
8. have fun!

2 Problem Set

1. **[6 Points]** LP or Not?
   For each of the following examples, determine whether the example is a linear programming problem. If not, why not?

   (a) **[2 Points]**

$$\begin{array}{ll} \text{maximize} & 2x_1 + 7x_2 \\ \text{subject to} & 2x_1^2 + 6x_2 \leq 11 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{array} \tag{1}$$

   (b) **[2 Points]**

$$\begin{array}{ll} \text{minimize} & 4x_1 + 6x_2 + 4^2 x_1 \\ \text{subject to} & 23x_1 + 5x_1 \leq 11x_1 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{array} \tag{2}$$

   (c) **[2 Points]**

$$\begin{array}{ll} \text{maximize} & 9x_1 + 3x_1x_2 \\ \text{subject to} & 27x_1 + 6x_1 \leq 25 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{array} \tag{3}$$

2. **[14 Points]** Warm up.
   Implement the following problems in Java/JOpt, find an optimal solution, report this solution and the value of the objective function at the solution.

   (a) **[4 Points]** Find a solution.

$$\begin{array}{ll} \text{minimize} & 2x_1 + 4x_2 \\ \text{subject to} & x_1 + x_2 \geq 3 \\ & 3x_1 + 2x_2 = 14 \\ & x_1 \geq 0 \end{array} \tag{4}$$

   (b) **[6 Points]** Two steps!

      i. **[4 Points]** Find a solution.

$$\begin{array}{ll} \text{maximize} & 4x_1 + 2x_2 \\ \text{subject to} & x_1 + 2x_2 \leq 3 \\ & 2x_1 + x_2 \leq 3 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{array} \tag{5}$$

      ii. **[2 Points]** Is this solution unique? Support your answer graphically!

(c) **[4 Points]** Find a solution.

$$\begin{array}{ll} \text{maximize} & x_1 + 0.64x_2 \\ \text{subject to} & 50x_1 + 31x_2 \leq 250 \\ & 3x_1 - 2x_2 \geq -4 \\ & x_1 \geq 0 \text{ integer} \\ & x_2 \geq 0 \text{ integer.} \end{array} \tag{6}$$

3. **[20 Points]** Trucks and Shipments.
A company produces three different products (called "1", "2", and "3") and ships them to its customer trough its 20 trucks. Assume that any product can be manufactured and sold in fractional quantities. The characteristics of each of them are reported in the table below.

Product	Raw material p.u.	Market value p.u.	Time p.u.	Truck p.u.
1	3	10	30	25%
2	4	12	45	10%
3	3	9	45	40%

In particular, given a certain product:

- 'Raw material p.u.' represents the cost of the raw material for producing 1.0 units of the product;
- 'Market value p.u.' represents the price in CHF at which 1.0 units of the product can be sold;
- 'Time p.u.' represents the time required to produce 1.0 units of the product in minutes;
- 'Truck p.u.' represents the weight of 1.0 units of the product, which is expressed as a percentage of the maximum weight capacity of a truck (each truck has the same maximum weight capacity).

Assume that each truck has a cost of transportation equal to

5 CHF x percentage of loaded weight

e.g., if the truck has a load equal to 50% of its maximum weight capacity, the cost for the transportation is 2.5 CHF. Furthermore, assume that the production can last no more than 90 hours. All produced (and shipped) units are bought by the costumer at market price.

What are the (fractional) quantities of each product that our company should produce in order to maximize its revenue?

(a) **[7 Points]** Formulate this problem as a linear programming problem. Be clear about what the variables are, what the objective function is, and what the constraints are.

(b) **[4 Points]** Implement the linear program in Java/JOpt, solve it, and report an optimal solution and the value of the objective function at the solution. What can you observe about product 3?

(c) **[2 Points]** Assume now that the company wants to have a balanced production, i.e., each single item has to cover at least 20% of the total production (in terms of units produced). Formulate the new linear programming problem, implement it in Java/JOpt, and report an optimal solution and the value of the objective function at the solution.

(d) **[7 Points]**Assume that the company can borrow up to 10 'smart' trucks for the shipment of its products, each of them with a cost of transportation equal to

2 CHF x percentage of loaded weight,

these trucks having the same weight tolerance of the ones belonging to the company. Formulate the new linear programming problem, implement it in Java/JOpt, and report an optimal solution and the value of the objective function at the solution. How many smart trucks should the company rent?
(Note: the company still wants to have a balanced production!)