## 11. Recursion

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## Objectives

- become familiar with the idea of recursion
- learn to use recursion as a programming tool $\qquad$
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## Introduction to Recursion

- A recursive algorithm will have one subtask that is a small version of the entire algorithm's task
- A Java method definition is recursive if it contains an invocation of itself.
- The method continues to call itself, with ever simpler cases, until a base case is reached which can be resolved without any subsequent recursive calls.


## Example: Exponent

```
private int power(int x, int y) {
    // y>=0 returns }\mp@subsup{x}{}{\wedge}
}
3)
\(x^{y}=1\) * \(x^{*} x^{*} \ldots\) * \(x\) ( \(y\) times)
- if \(y==0\), then stop and return 1
if \(y>0\), then multiply \(x\) with the result of \(x^{(y-1)}\)
```


## Exponent/2

$\qquad$
private int power(int $x$, int $y$ ) \{
/ $y>=0$ returns $x^{* *} y$
if ( $\mathrm{y}==0$ )
return 1
int assistantResult $=\operatorname{power}(x, y-1)$
return x * assistantResult;
$\}^{\}}$


## Activation records

- f() calls power(3, 2):



## Stack of Activation records /2

- After power $(3,1)$ has been called:

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Stack of Activation records /3 $\qquad$

- After power $(3,0)$ has been called



## Return

A return-Statement

- evaluates the return value (e.g., 1)
- deletes the current activation record $\qquad$
replaces the expression that called the method with the return value
. continues execution of the caller $\qquad$

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$\qquad$



## Return /3


$\qquad$
Sender: method f, line N Activation record for power (3,2) (the current activation record $\qquad$
$\qquad$
Sender: whoever invoked $f$
Activation record for $\mathrm{f}($. ...)
(the current activation record)

## Example: Digits to Words

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- Write a definition that accepts a single integer and produces words representing its digits.
- Example
$\qquad$
output: two two three
$\qquad$
$\qquad$
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## Digit to Words: Specification

If number has multiple digits, decompose algorithm into two subtasks

- Display all digits but the last as words
- Display last digit as a word

First subtask is smaller version of original problem

- Same as original task, one less digit


## Recursion Guidelines

- The definition of a recursive method typically includes an if-else statement.
- One branch represents a base case which can be solved directly (without recursion).
- Another branch includes a recursive call to the method, but with a "simpler" or "smaller" set of arguments.
- Ultimately, a base case must be reached (termination).


## Termination

- You need to have a return-statement that does not make a recursive call
- The return statement needs to be before the recursive call
if ( $y==0$ )
return 1
else $\{$....
\}
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## Infinite Recursion

- If the recursive invocation inside the method does not use a "simpler" or "smaller" parameter, a base case may never be reached.
- Such a method continues to call itself forever (or at least until the resources of the computer are exhausted as a consequence of stack overflow)
- This is called infinite recursion


## Infinite Recursion

- Suppose we leave out the stopping case
public static void displayAswords(int number)/Not quite right
displayAsWords (number / 10);
System.out.print(getWord FronDigit(number \% 10) + " ");
\}
- Nothing stops the method from repeatedly invoking itself
- Program will eventually crash when computer exhausts its resources (stack overflow)


## Recursive Versus Iterative

- Any method including a recursive call can be rewritten to do the same task without recursion
- Recursive method
- Uses more storage space than iterative version
- Also runs slower
- However in some programming tasks, recursion is a better choice, a more elegant solution
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## Overloading is Not Recursion

- If a method name is overloaded and one method calls another method with the same name but with a different parameter list, this is not recursion
- Of course, if a method name is overloaded and the method calls itself, this is recursion
- Overloading and recursion are neither synonymous nor mutually exclusive


## Example: Family Tree



## Summary

- To avoid infinite recursion recursive method should contain two kinds of cases
- A recursive call
- A base (stopping) case with no recursive call
- Good examples of recursive algorithms
- Binary search algorithm
- Merge sort algorithm
- Operations in tree structures

