University of Zurich[UZH]

Department of Informatics

University of Zürich
Department of Informatics
Binzmühlestr. 14
CH-8050 Zürich
Phone. +41 44 635 43 11
Fax +41 44 635 68 09
www.ifi.uzh.ch/dbtg

UZH, Dept. of Informatics, Binzmühlestr. 14, CH-8050 Zürich

Prof. Dr. Michael Böhlen
Professor
Phone +41 44 635 43 33
Fax +41 44 635 68 09
boehlen@ifi.uzh.ch

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MSc Basic Module
Title: Identifying desirable spanning trees for dynamic connectivity

In undirected fully-dynamic graphs, the connectivity of two vertices are consistently changing due to the insertion and deletions of edges. (Forests of) spanning tree can be used to maintain the connected components of a graph. For an undirected graph, there are many different spanning trees. Among all spanning trees of a component, some spanning trees show better performance when it comes to updating and querying them. When a spanning tree is unevenly split – a small spanning tree and a large spanning tree – it becomes faster to go through the smaller spanning tree to find an edge that potentially reconnects the split spanning trees (if it exists).

To formalize this approach, we introduce the cut number of an arc e in a spanning tree, denoted as $c(e)$. After the cut of an arc e of the spanning tree, two spanning trees are generated $tree_1$ and $tree_2$; $c(e) = min(size(tree_1), size(tree_2))$, where $size(tree_1)$ and $size(tree_2)$ are the number of nodes in $tree_1$ and $tree_2$, respectively. Assume that E_{st} is the set of arcs of the spanning tree st, our goal is to find spanning trees that minimize average cut number.

$$average\ cut\ number = \frac{1}{|E_{st}|} \sum_{e \in E_{st}} c(e)$$

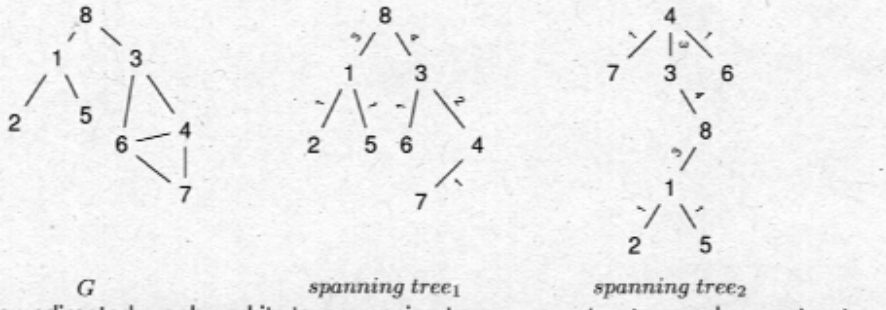
This Master Basic Module aims to find spanning trees that minimize the average cut number.

Tasks:

- **Task 1: Study by examples**
  - Study the illustration to understand basic concepts.
- **Task 2: brute-force approach**
  - Given an undirected graph, enumerate all the spanning trees.

  - Calculate average cut number for all spanning trees.
  - Find the spanning trees with minimum average cut number.
- **Task 3: Study properties of the desirable spanning tree**
  - Investigate the spanning trees with minimum average cut number to summarize their properties.
  - Report your findings.
- **[optional] Task 4: Improved data structures and algorithms**
  - Using dynamic tree structures to find spanning trees with minimum average cut number.
  - Develop efficient algorithms that construct spanning tree with minimum average cut number.
  - Investigate how to design update operations on spanning trees that keep the cut number low.
- **Task 5: Write a brief report**
  - Summarize all the findings in a short report of about eight to ten pages.

In figure 1, for the input undirected graph G, two spanning trees $spanning\ tree_1$ and $spanning\ tree_2$ are shown. In $spanning\ tree_1$ and $spanning\ tree_2$, the values on the arcs are their cut numbers. For $spanning\ tree_1$, $average\ cut\ number = \frac{13}{7}$. For $spanning\ tree_2$, $average\ cut\ number = \frac{14}{7}$.



G $spanning\ tree_1$ $spanning\ tree_2$

figure 1: An undirected graph and its two spanning trees: $spanning\ tee_1$ and $spanning\ tee_2$

Supervisor: Sven Helmer (helmer@ifi.uzh.ch), Qing Chen (qing@ifi.uzh.ch)

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University of Zurich
Department of Informatics

i.V. Sven Helmer

Prof. Dr. Michael Böhlen
Professor