Business Network Analytics and Business Intelligence

Network Modeling & Analysis

Financial Markets, Banking Systems...

Risk Contagion (e.g., Bank run)

Economic Networks

Monitoring, Analyzing, and Simulating Contagious Risks, ...

Design

BI Applications

Identification and Prediction of Influence-Driven Network Diffusion

Social Networks

Online Communities, Consumer Social Networks, ...

Social Contagion (e.g., Word-of-Mouth)
During 2008 financial tsunami, which bank(s) should we inject capital first to stop contagious failures in bank networks?
Utilize Peer Influence in Online Social Networks

- Influencer Marketing, Product Recommendation
  - Who are the most influential people?
  - What are the patterns of information diffusion?
Develop Strategies to Attack Terrorist Networks

A Global Salafi Jihad Terrorist Network
Hu et al. JHSEM 2009

- How to effectively break down a terrorist network?
Business Networks Analytics and Applications

• Instructor: Prof. Dr. Daning Hu, BIN. 2.A.12
• TA: Xiao Li, BIN.2.A.24
• Email: hdaning@ifi.uzh.ch
• Credits: 5 ECTS credits
• Class Schedule:
• Language: English
• Audience: Undergraduate and Master students
• Office Hours: Email for appointment, Room 2.A.12.
Grading and Course Goals

• 1. One course project (90%)

• 2. Active participation and interaction during the lectures and tutorials (10%)

• The project report should include the following four major components:
  – **Network/Relational Data Collection** (15%)
  – **Network Data Processing and Modeling** (20%)
  – **Network Visualization** (15%)
  – **Network Analysis** (30%)
Example 1: Network Data Collection

- Social Networks: Online communities, Social networking websites, Personal blogs and micro-bloggings, online video sharing websites. (e.g., Programmable Web)


- Others: Financial, Education data sources: Stanford SNAP Portal
Example 2: Network Data Processing Modeling

- Extract relations/links from raw data in database tables

<table>
<thead>
<tr>
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<th>Minor diameter tolerance</th>
<th>Nominal diameter (mm)</th>
<th>Head shape</th>
<th>Price for 50 screws</th>
<th>Available at factory outlet?</th>
<th>Number in stock</th>
<th>Flat or Phillips head?</th>
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</thead>
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</table>

- Model such relations/links into network data.
  - Node data
  - Link data
Example 3: Network Visualization
Example 4: Network Analytics

A Microblogging Network: Who possesses the most advantageous position in brokering information and knowledge in this network?
Example 5: A Global Terrorist Network

- How to effectively break down terrorist networks?
Business Network Analytics or Applications

- Recommender Systems:

  Recommended for You
  
  se recommendations are based on items you own and more.

  Hitman (2007)
  You rated this movie: 3.0 stars
  Average of 94,356 ratings: 3.0 stars
  
  Details
  
  unrated. This movie has not been rated by the MPAA
  PARENTS: G for 16+ (more)

  More Like This
  
  Here are some other movies you might enjoy...
  
  Hitman
  The House of Black
  Don Whiplash
  cucumber Million: Season 2
  (8-Disc Series)
Business Network Analytics or Applications

- Social Media based Marketing, Word-of-Mouth Effect
Computing Tools Required In Tutorials

• Database Management Software
  – MySQL or other common DBMS such as MS SQL Server, Oracle, etc.

• Network Visualization Tool: NetDraw, R (Statnet or iGraph)

• Network Analysis Tool: R (Statnet or iGraph), or UCINet
Outline

- Syllabus
- Examples
- Introduction
- Social Network Analysis
- Social Network Data Modeling and Analysis

* Some of the contents are adapted from Prof. James Mody’s slides at Duke University, and Prof. Jure Leskovec and Lada Adamic from Stanford University.
Introduction: Why Study Networks?

- One of the most profound changes in today’s world is - **Decentralization**
  - Economical: BitCoin, Blockchain, P2P lending, etc.
  - Social: Social Media News, Online Communities, Terrorist Cells, etc.
  - Technological: Open Source Software, Virtual Teams, etc.

- The power, information, resources in real world networks are becoming increasingly decentralized ->
  - Nodes are distributed more equally, Less hierarchical;
  - Good representations of entities and relationships in decentralized systems.
The Focus: The Influence Mechanisms in Networks

- An average individual (node) can affect system outcome by **influencing** its linked peers.

- This course will focus on the “**influence**” mechanisms that network actors affect each other in terms of opinions, behaviors, or risks through various ties, thereby changing the network outcomes:
  - Economic influence: Risk contagion, etc.
  - Social influence: Word-of-Mouth, observation learning, herding, etc.
  - Network outcomes: Bank run, stock market crash, product diffusions.
A “Random” History of Network Science

1736
- **Mathematical** foundation – Graph Theory

1930
- **Social** Network Analysis and Theories
  - Sociogram: Network visualization
  - Six degree of separation
  - Structural hole: Source of innovation

1990
- **(Physicists)** Complex Network Topologies
  - Small-world model (e.g., WWW)
  - Scale-free model (“Rich get richer”)

2000
- **Network Science**
  - Economic networks (Agent modeling & simulation)
  - Dynamic network analysis
  - BI applications: product diffusion in social media, recommendation systems

2017
- ?
Network Science

• Network science is an **interdisciplinary** academic field which studies complex networks such as information networks, biological networks, cognitive and semantic networks, and **social networks**. It draws on theories and methods including:
  
  – **Graph theory** from mathematics, e.g., Small-world
  
  – **Statistical mechanics** from physics, e.g., Rich get richer,
  
  – **Data mining** and **information visualization** from computer science,
  
  – **Inferential modeling** from statistics, e.g., Collaborative filtering
  
  – **Social structure** from sociology, e.g., weak tie, structural holes

• Network science can be defined as "**the study of network representations of physical, biological, and social phenomena leading to predictive models of these phenomena.**"
The study of networks has emerged in diverse disciplines as a means of analyzing complex relational data.

Network science has its root in **Graph Theory**.
- Seven Bridges of Königsberg written by Leonhard Euler in 1736.
- Focusing on the properties of pairwise relations in a network structure.

Social Network Analysis
- Jacob Moreno, a psychologist, developed the Sociogram and to “precisely describe the interpersonal structure of a group”.
“For the last thirty years, empirical social research has been dominated by the sample survey. But as usually practiced, ..., the survey is a sociological meat grinder, tearing the individual from his social context and guaranteeing that nobody in the study interacts with anyone else in it.”

*Allen Barton, 1968 (Quoted in Freeman 2004)*

Moreover, the complexity of the relational world makes it impossible to identify social connectivity using only our intuition.

Social Network Analysis (SNA) provides a set of tools to empirically extend our theoretical intuition of the patterns that compose social structure.
Social network analysis (SNA) is a set of \textit{relational} methods for systematically understanding and identifying connections /ties /relationships among actors.

Social network analysis (SNA)
- is motivated by a structural intuition based on ties linking social actors
- is grounded in systematic empirical data
- draws heavily on graphic theory and imagery
- relies on the use of mathematical and computational models.
Jacob Moreno’s experiment on Friendship Network

Jacob’s experiment is the first to use Social Network Analysis
Social Network analysis lets us answer questions about social interdependence. These include:

“Networks as Variables” approaches
- Are kids with smoking peers more likely to smoke themselves?
- Do unpopular kids get in more trouble than popular kids?
- Do central actors control resources?

“Networks as Structures” approaches
- What generates hierarchy in social relations?
- What network patterns spread diseases most quickly?
- How do role sets evolve out of consistent relational activity?

We don’t want to draw this line too sharply: emergent role positions can affect individual outcomes in a ‘variable’ way, and variable approaches constrain relational activity.
Now…

Complex Networks in the Real World

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social network</td>
<td>Friendship, kinship, collaboration</td>
</tr>
<tr>
<td>Inter-organizational network</td>
<td>Strategic alliance, buyer-seller relation, joint venture</td>
</tr>
<tr>
<td>Citation network</td>
<td>Citations</td>
</tr>
<tr>
<td>Internet</td>
<td>Wire, cable</td>
</tr>
<tr>
<td>WWW</td>
<td>hyperlink</td>
</tr>
<tr>
<td>Biochemical network</td>
<td>Regulatory effect</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Now…

• Universal modeling and analysis methods for complex network data

• Shared vocabulary between fields: Computer Science, Physics, Sociology, Economics, Statistics, Biology

• “Big” Data availability: Internet, mobile, bio, health, security…

• Impact/usage: social networking, social media, marketing, etc.
## Our Approach

### What
- Social network analysis (Metrics)
- **Describe** the changes in network evolution
  - Temporal changes in network topological measures
- Dynamic network recovery
- (Relational) data mining

### Why
- Econometric **identification** of casual Social and Economic influence
  - Distinguishing homophily
  - Confounding factors
  - PSM, DID, RD, etc.
  - Explanations

### How
- **Combine** social science methods, data mining, machine learning with econometric analysis
- **Predict** link formation
- **Simulate** the evolution of networks
What: Social Network Analysis

- Social network analysis (SNA) is a set of **metrics** and methods for systematically **describing**, modelling, and analyzing relationships among actors.

- Social network analysis (SNA)
  - is motivated by a structural intuition based on ties linking social actors
  - is grounded in systematic empirical data
  - draws heavily on graphic imagery
  - relies on the use of mathematical and/or computational models.
What is a Network?

**Actor/Node:** Any entity in a network (person, system, group, organization)

**Tie/Link:** Relationship or interaction between two nodes.
Basic Concepts in (Social) Network Analysis

- Node, Actor, Vertex \( V \)
- Tie, Link, Edges \( E \)
- Network, System \( G (V, E) \)

A link can be (1) Binary or Valued, (2) Directed or Undirected.
Nodes or Social Actors

- Social Network data consists of two linked classes of data: Nodes and Links.

- Node Example: Products in a purchase network

- Actor Examples: people in a group, departments within in a corporation, public service agency in a city, nation-states in the world system. “Node” does not imply that they have intention or the ability to “act”.

- Network nodes are most often people, but can be any other unit capable of being linked to another (schools, countries, organizations, personalities, etc.)
Links or Ties

- Actors (nodes) are linked to one another by social ties (links)
  - Kinship, role-based, cognitive, affective, interactions, affiliations

- Example of **direct** links in SNA (Wasserman/Faust 2008:17):
  - Evaluation of one person by another (friendship, liking, or respect)
  - Transfers of material resources (business transactions, lending or borrowing things)
  - Behavioral interaction (talking together, sending messages)
  - Physical connection (a road, river, or bridge connecting two points)

<table>
<thead>
<tr>
<th>Similarities</th>
<th>Social Relations</th>
<th>Interactions</th>
<th>Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Kinship</td>
<td>e.g., Sex with</td>
<td></td>
</tr>
<tr>
<td>e.g., Same</td>
<td>e.g., Mother of</td>
<td>Talked to</td>
<td>e.g.,</td>
</tr>
<tr>
<td>spatial and</td>
<td>Sibling of</td>
<td>Advice to</td>
<td>Information</td>
</tr>
<tr>
<td>temporal space</td>
<td></td>
<td>Helped</td>
<td>Beliefs</td>
</tr>
<tr>
<td>e.g., Same</td>
<td></td>
<td>Harmed</td>
<td>Personnels</td>
</tr>
<tr>
<td>events</td>
<td>Other role</td>
<td>etc.</td>
<td>Resources</td>
</tr>
<tr>
<td>e.g., Same</td>
<td>e.g., Friend of</td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>gender</td>
<td>Boss of</td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>e.g., Same</td>
<td>Student of</td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>attitude</td>
<td>Competitor of</td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>etc.</td>
<td>Affective</td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>e.g., Likes</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>Hates</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>etc.</td>
<td>Cognitive</td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>e.g., Knows</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>about</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>Sees as</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>happy</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td></td>
<td>etc.</td>
</tr>
</tbody>
</table>
Weight/Strength of Ties

- We can attach values to ties, representing quantitative attributes
  - Strength of relationship
  - Information capacity of tie
  - Rates of flow or traffic across tie
  - Distances between nodes
  - Probabilities of passing on information
  - Frequency of interaction
- Valued graphs or vigraphs
Positive and Negative Weights

- e.g. one person trusting/distrusting another

- Research challenge: How does one ‘propagate’ negative feelings in a social network? Is my enemy’s enemy my friend?

*Sample of positive & negative ratings from Epinions network*
## Two Modes of Social Network Analysis

<table>
<thead>
<tr>
<th></th>
<th>Complete</th>
<th>Ego</th>
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<tbody>
<tr>
<td><strong>1-mode</strong></td>
<td><a href="#">Diagram</a></td>
<td><a href="#">Diagram</a></td>
</tr>
<tr>
<td>Bill → Bob → Biff → Betty</td>
<td><a href="#">Diagram</a></td>
<td><a href="#">Diagram</a></td>
</tr>
<tr>
<td><strong>2-mode</strong></td>
<td><a href="#">Diagram</a></td>
<td><a href="#">Diagram</a></td>
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<tr>
<td><a href="#">Diagram</a></td>
<td><a href="#">Diagram</a></td>
<td><a href="#">Diagram</a></td>
</tr>
</tbody>
</table>

### 1-mode
- **Bill**: Connected to **Bob** and **Betty** through **Biff**.
- **Bob**: Connected to **Biff** and **Betty**.
- **Betty**: Connected to **Biff**.

### 2-mode
- **Dr. Jones**: Connected to **Web MD**.
- **Bill**: Connected to **PDR**.
- **Patient**: Connected to **Mom**, **Jane**, and **Merck manual**.
One-mode Complete network

Information flow within virtual group

Data collected by Cross
One-mode Ego network

Year 1

Data courtesy of Michael Link

Year 4
Ego Network Analysis combine the perspective of network analysis with the data of mainstream social science.

No computer assisted analysis needed.
Two-mode Complete Network (Bipartite Graph)

Data compiled from newspaper society pages by Davis, Gardner & Gardner
Two-mode (Bipartite) Network Transformation

From Zan Huang et al., 2009, Management Science
Network Data Modeling: Adjacency Matrix

### Friendship

<table>
<thead>
<tr>
<th></th>
<th>Jim</th>
<th>Jill</th>
<th>Jen</th>
<th>Joe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim</td>
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<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Jill</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Jen</td>
<td>0</td>
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<td>-</td>
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</tr>
<tr>
<td>Joe</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

### Proximity

<table>
<thead>
<tr>
<th></th>
<th>Jim</th>
<th>Jill</th>
<th>Jen</th>
<th>Joe</th>
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<td>3</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Jill</td>
<td>3</td>
<td>-</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>Jen</td>
<td>9</td>
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<td>-</td>
<td>3</td>
</tr>
<tr>
<td>Joe</td>
<td>2</td>
<td>15</td>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>
### Network Distance (Weighted) Adjacency Matrix

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
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<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
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<td>e</td>
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<td>2</td>
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<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>g</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0</td>
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</table>
# Major Network Data Formats (in UCINet)

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<th>Labels</th>
<th>Data</th>
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<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
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<td>embedded</td>
<td></td>
</tr>
</tbody>
</table>

**DI n = 5**

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<tbody>
<tr>
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<td></td>
</tr>
<tr>
<td><code>nodelist</code></td>
<td></td>
</tr>
<tr>
<td><code>edgelist</code></td>
<td></td>
</tr>
</tbody>
</table>

**Data: Billy, jill, john, jim, jane**

<table>
<thead>
<tr>
<th></th>
<th>billy</th>
<th>john</th>
<th>jill</th>
<th>mary</th>
</tr>
</thead>
<tbody>
<tr>
<td>billy</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>john</td>
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**Data: Billy, jill, john, bertha**

<table>
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**Data: Billy, jill, john, 6.3, jane**

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**Data: Billy, jill, john, 2.5**

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<tr>
<td>mary</td>
<td>1</td>
<td>0</td>
<td>1</td>
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</tbody>
</table>

**Note:**
- **Values optional**
- **Values optional - assigned 1 if omitted**
- **No values possible**
Real World Networks are Sparse Graphs

Most real-world networks are sparse
\[ E \ll E_{\text{max}} \quad \text{(or } \bar{k} \ll N-1) \]

<table>
<thead>
<tr>
<th>Network</th>
<th>N</th>
<th>\langle k \rangle</th>
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</thead>
<tbody>
<tr>
<td>WWW (Stanford-Berkeley)</td>
<td>319,717</td>
<td>9.65</td>
</tr>
<tr>
<td>Social networks (LinkedIn)</td>
<td>6,946,668</td>
<td>8.87</td>
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<tr>
<td>Communication (MSN IM)</td>
<td>242,720,596</td>
<td>11.1</td>
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<tr>
<td>Coauthorships (DBLP)</td>
<td>317,080</td>
<td>6.62</td>
</tr>
<tr>
<td>Internet (AS-Skitter)</td>
<td>1,719,037</td>
<td>14.91</td>
</tr>
<tr>
<td>Roads (California)</td>
<td>1,957,027</td>
<td>2.82</td>
</tr>
<tr>
<td>Proteins (S. Cerevisiae)</td>
<td>1,870</td>
<td>2.39</td>
</tr>
</tbody>
</table>

(Source: Leskovec et al., Internet Mathematics, 2009)

Consequence: Adjacency matrix is filled with zeros!
(Density of the matrix \( \frac{E}{N^2} \): WWW = 1.51 \times 10^{-5}, MSN IM = 2.27 \times 10^{-8})
More Types of Networks

- **Self-edges (self-loops)** (undirected)
  
  
  \[
  A_{ij} = \begin{pmatrix}
  1 & 1 & 1 & 0 \\
  1 & 0 & 1 & 1 \\
  1 & 1 & 0 & 0 \\
  0 & 1 & 0 & 1
  \end{pmatrix}
  \]

- **Multigraph** (undirected)
  
  
  \[
  A_{ij} = \begin{pmatrix}
  0 & 2 & 1 & 0 \\
  2 & 0 & 1 & 3 \\
  1 & 1 & 0 & 0 \\
  0 & 3 & 0 & 0
  \end{pmatrix}
  \]