Lecture #3: Game Theory II: Extensive-Form and Repeated Games

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Housekeeping

- Questions?
- Concerns?
- Piazza → All announcements from now on.
- NB?
  - Ask questions visible to the class
  - Don't delete questions, but answer them yourself.
- Homework assignments?
- Grading policy

Outline

1. Recap of last lecture
2. Game Theory II
3. Play some in-class experiments
4. Discussion
5. Questions

Quick Recap

- PD Game
  - Payoffs
  - Utilities and affine transformations
- Pareto-optimality:
  - Definition
  - what should the prisoner do? (Q1)
- Where do we have PD games in real life?

- Dominant Strategies
- Iterated elimination of dominated strategies
- Nash equilibrium
  - Strategies vs. outcomes!
  - Pure strategies
  - Mixed strategies (how do I see one is a NE)
  - Matching Pennies: why no pure-Nash? (Q2)
  - Multiplicity

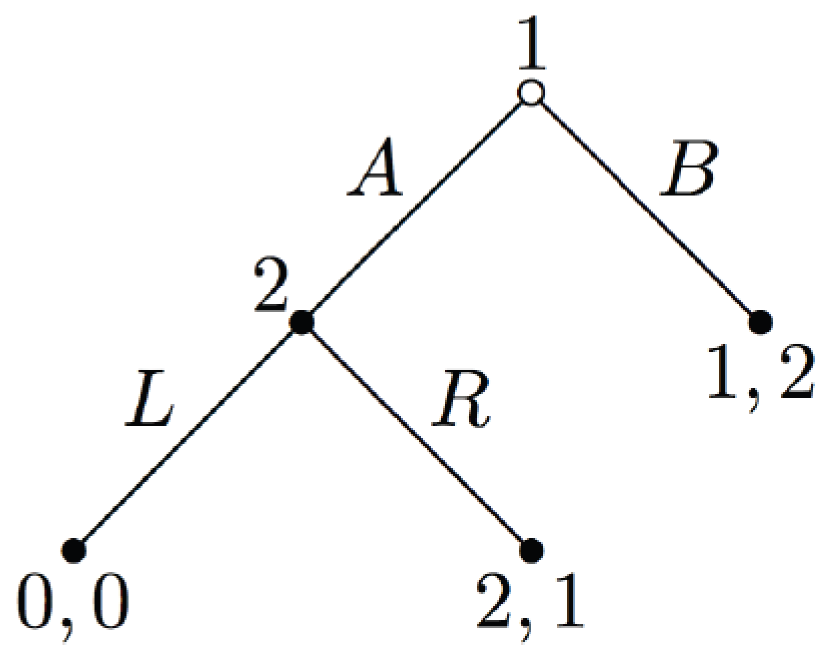
Prisoner's Dilemma Game

		Player 2	
		C	D
Player 1	C	3, 3	0, 5
	D	5, 0	1, 1

Topics for Today

- Extensive form games:
  - Tree notation, terminal nodes
  - Strategy in extensive form games
  - Nash equilibrium in extensive form games
  - What do players know?
  - Incredible threats
  - Subgames
  - Subgame-perefect NE
  - Principle of one deviation
  - Backwards induction
  - Centipede Game
- Repeated games:
  - Discounted sum
  - Automaton Strategies (4 examples)
  - Open Loop Strategies (example with 3player game of chicken)
  - Folk Theorems
    - Feasible payoffs
    - Enforceable payoffs

Extensive-Form Games



Strategies in Extensive-Form Games

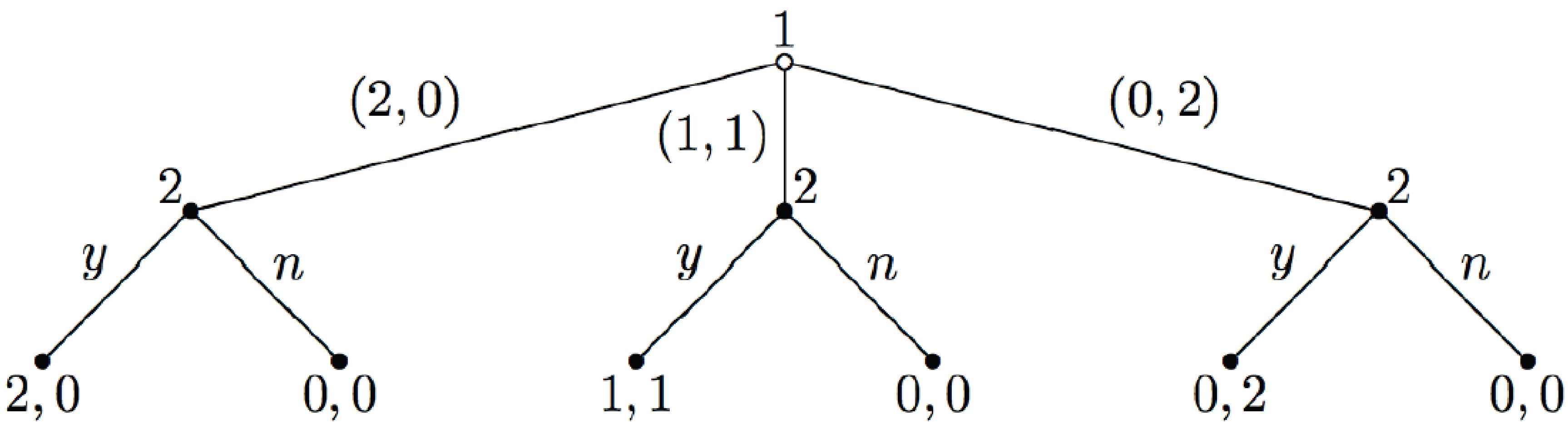
Definition 3.2 (strategy in extensive-form game)**.** *A pure* strategy s_i *in an extensive-form game assigns an action in* $\phi_i(h)$ *to every non-terminal history* $h \in H$ *for which* $i = P(h)$*.*

Nash-Equilibrium in Extensive-Form Games

Definition 3.3 (Nash equilibrium in extensive-form game)**.** *A strategy profile* $s^* = (s_1^*, \ldots, s_n^*)$ *is a (pure-strategy) Nash equilibrium of an extensive-form game if, for all* i*,*

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \quad \text{for all } s_i \in S_i \tag{3.1}$$

Bargaining Game



Subgame-perfect Nash Equilibrium

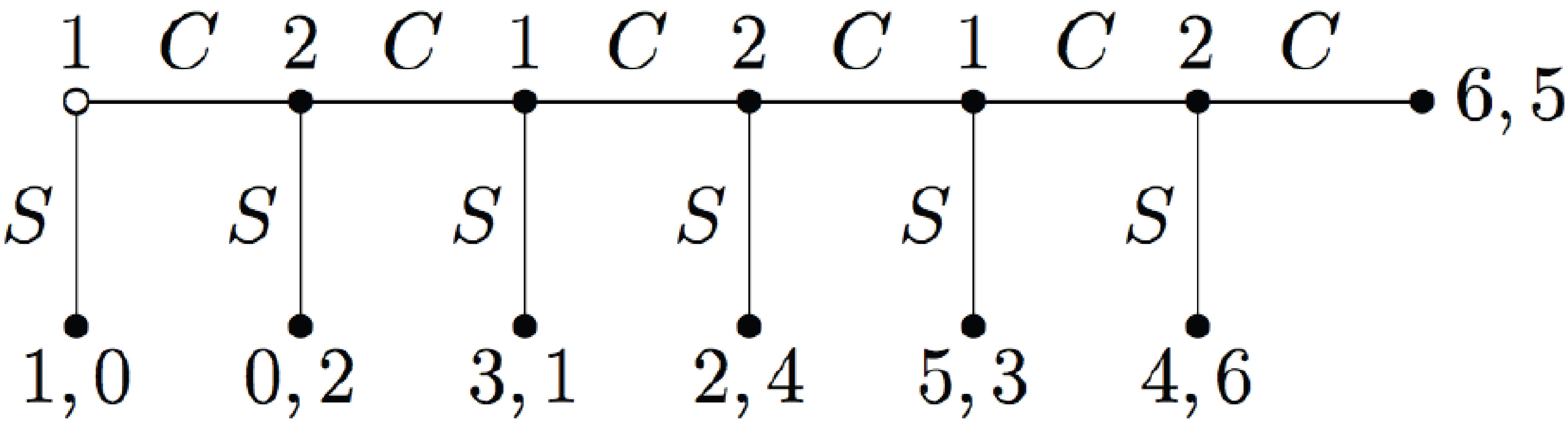
Definition 3.5 (subgame-perfect equilibrium). *A strategy profile* $s^* = (s_1^*, \ldots, s_n^*)$ *is a subgame-perfect equilibrium of an extensive-form game if the strategy profile is a Nash equilibrium in the subgame at every non-terminal* $h \in H$.

Principle of one deviation

Theorem 3.1 (principle of one deviation). *Strategy profile s^* is a subgame-perfect equilibrium in a finite extensive-form game if and only if, for the subgame defined at every non-terminal history $h \in H$, agent $i = P(h)$ has no useful deviation when changing its action in the play at that history h only.*

→ Backwards Induction Algorithm…

The Centipede Game



Let’s play it!

- Students enter '1-8', with 1,3,5,7 meaning they are player 1, and 2,4,6,8 meaning they are player 2.

Repeated Games

Definition 3.6 (repeated game)**.** *Given a normal-form game* $G = (N, A, u)$*, a repeated game is an extensive-form game in which the stage game* G *is repeated (finitely or infinitely), and every agent plays in every round, with perfect information about the history (and thus the sequence of action profiles in all previous rounds.)*

Example: Repeated PD

		Player 2 C	Player 2 D
Player 1	C	3, 3	0, 5
	D	5, 0	1, 1

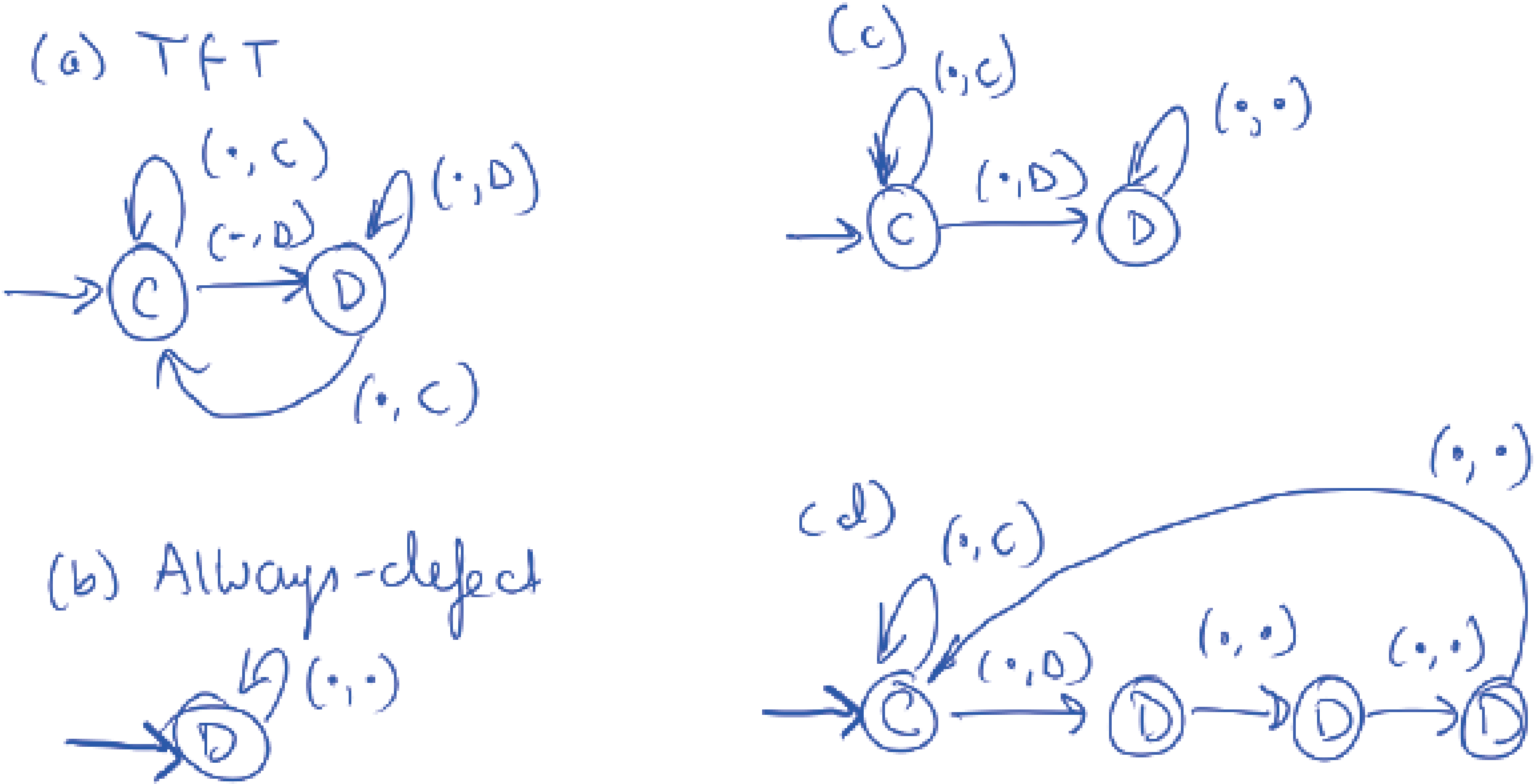
- Discounted Utility: $u_i(h) = \sum_{k=0}^{\infty} \delta^k u_i(a^k)$

Automaton Strategies

Definition 3.7 (automaton strategy). *An automaton strategy m_i for player i in a repeated game is defined by $(Q_i, q_i^0, succ_i, f)$ where:*

- *Q_i is a set of states*[7]
- *$q_i^0 \in Q_i$ is the start state*
- *$q_i' = succ_i(q_i, a)$ is the next state q_i', given state q_i and action profile $a = (a_1, \ldots, a_n) \in A$, for joint action state A*
- *$f_i(q_i) \in A_i$ prescribes an action for agent i in state q_i*[8]

Sample Automata



Feasible and Enforceable

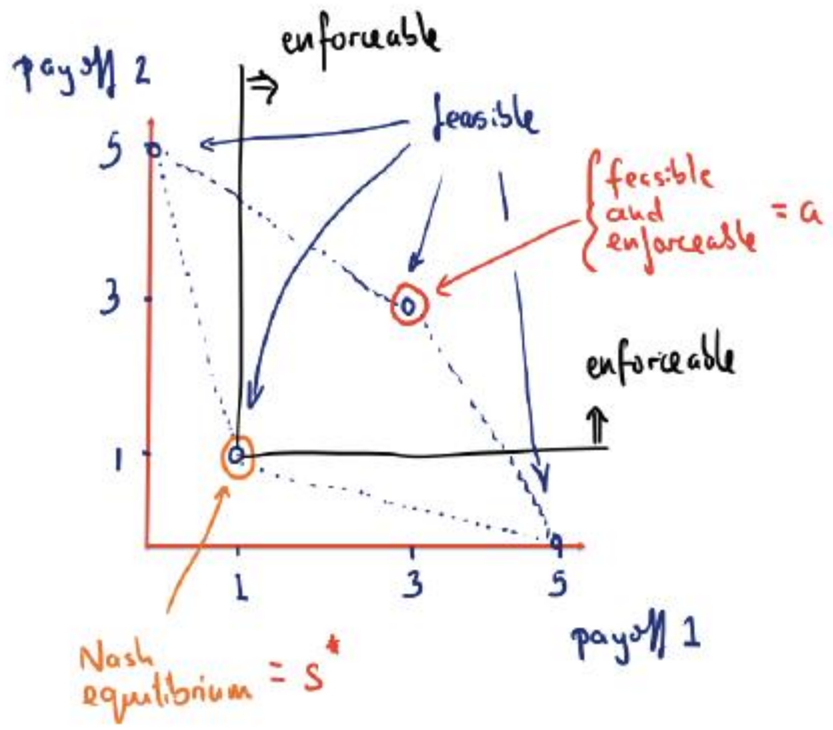


Figure 3.8: Feasible and Enforceable payoffs in the *Prisoner's Dilemma.*

Folk Theorem

Theorem 3.6 (Folk theorem). *Given a stage game G with feasible and enforceable payoff profile $v = (v_1, \ldots, v_n)$, then for some $\delta < 1$ there exists a subgame-perfect equilibrium of the infinitely repeated game G^* that provides discounted utility $\frac{v_i}{1-\delta}$ to every agent i.*

Open-Loop Strategies…