

**Lecture #7.1:
Sponsored Search Auctions (Part I)**

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Housekeeping

- Questions? Concerns?
- BitTorrent homework:
 - due now
 - What took so long?
- AGT/Auctions homework:
 - will be online tonight
 - Proofs...
- Book/cancel course (+CHF 50) until end of week
- Come to our office hours if you have questions
- Start early with assignments/with asking questions
- English
 - one equilibrium, two equilibria!
 - advertiser → “ad” not “add”!



Outline

1. Recap of Auction Theory
2. Today's topic: Sponsored Search Auctions
3. Discussion + Questions

Recap: Auction Theory

- What's an auction
- Quasi-linear utility assumption
- 2nd-price Auction → Dominant-Strategy NE
- 1st-price Auction → Bayes-Nash-Equilibrium
- Efficiency, Revenue
- Revenue Equivalence Theorem
- eBay Auctions...

Today: Sponsored Search Auctions



Suche Ungefähr 195'000'000 Ergebnisse (0.21 Sekunden)

Alles

Bilder

Maps

Videos

News

Shopping

Mehr

Zürich
Standort ändern

Das Web
Seiten auf Deutsch
Seiten aus der Schweiz
Übersetzte Seiten

Alle
Letzte Stunde
Letzte 24 Std.
Letzte 2 Tage
Letzte Woche
Letzter Monat
Letztes Jahr
Zeitraum festlegen...

Alle Ergebnisse
Websites mit Bildern
Verwandte Suchanfragen
Mehr Optionen

Philips LED-Fernseher | philips.ch
www.philips.ch/fernseher
Vergleichen und Händler suchen auf der offiziellen Philips Website!

Plasma TV | brack.ch
www.brack.ch/Fernseher
Jetzt den richtigen **TV** finden Mit der Auswahlhilfe kein Problem.
148 Personen geben +1 für Brack.ch


Tipp: [Suchen Sie nur nach Ergebnissen auf Deutsch](#). Sie können Ihre Sprache in den [Einstellungen](#) festlegen.

TV Reviews: LCD TVs, flat-screen TVs, plasma TVs, 3D TVs - CNE...
reviews.cnet.com/televisions/ - Diese Seite übersetzen
Television reviews and ratings, video reviews, user opinions, most popular **tv**s, **tv** buying guides, prices, and comparisons from CNET Reviews.

Flat panel display - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/Flat_panel_display - Diese Seite übersetzen
The first engineering proposal for a **flat screen TV** was by General Electric as a result of its work on radar monitors. Their publication of their findings gave all the ...

LCD-TV Preisvergleich | LCD-TVs - Preise bei idealo.at
www.idealoo.at > TV, Video, DVD > Fernseher
LCD-TV: erst Preisvergleich dann kaufen. Insgesamt 1546 preiswerte LCD-TV's , davon 342 mit Tests (Stand 18.03.2012). Günstige Preise der Online-Shops bei ...

Bilder zu flat screen tv - Unangemessene Bilder melden



Preise Schweiz - Monitor-Flatscreen Preisvergleich, Preis und ...
www.preissuchmaschine.ch/kat2-28.html
1000 Produkte – Schweizer Preise für Monitor-**Flatscreen** Preisvergleich, Preis und Angebot. ... HOME Computer Monitor-**Flatscreen** ...

Plasma-TV
www.microspot.ch/
Unschlagbare Preise bei microspot!
Plasma-TV - Top-Preis

TV Möbel&Wandhalterungen
www.hifi-tv-moebel.ch/
Modernes Design **TV**-Möbel/Halterung, Zubehör. Kostenlos Versand

BeoVision 10 LCD TV
beovision10.com/de/beovision10-LCD
BeoVision 10 & Home-Styling im Gesamtwert von 14.300CHF gewinnen!

LED, LCD & Plasma TV Shop
www.digitaldiscount.ch/
alle **TV**s mit 4 Jahren CH-Garantie
20'000 Artikel zu Discountpreisen

LCD-TV
www.conrad.ch/conrad-garantie/
Auf **LCD-TV** bietet Conrad mindestens 2 Jahre Garantie. Immer! Alte Dübendorferstrasse 17
[Routenplaner](#)

Flat Screen TV Sale
www.zapmeta.de/
Alles zu **flat screen TV** sale
Flat Screen TV Sale - Mit Zapmeta

TV / HiFi Halterungen
www.beschlaege-online.ch/
elektr. **TV** Heber, **TV** Schwenkarme
Wand-, Deckenhalterungen etc.

Flatscreen TV bis -84%
www.dusparst.com/Flatscreen+TV
Beeilung! Nur heute 84% günstiger:
Flatscreen TV

Bidding Language

- Value per click $w_i \in [0, \infty]$
- Estimated click through rate for bidder i in slot j :
 $eCTR_{i,j} \in [0,1]$
- Effective value: $v_i(j) = \begin{cases} eCTR_{i,j} \times w_i & \text{if allocated} \\ 0 & \text{else} \end{cases}$
- Assumptions:
 - Separability: 1) bidder effect (quality) and 2) slot effect:
 $eCTR_{i,j} = q_i \times s_j$ [Q1]
 - Decreasing slot effect: $s_1 \geq s_2 \geq s_3 \dots \geq s_m$

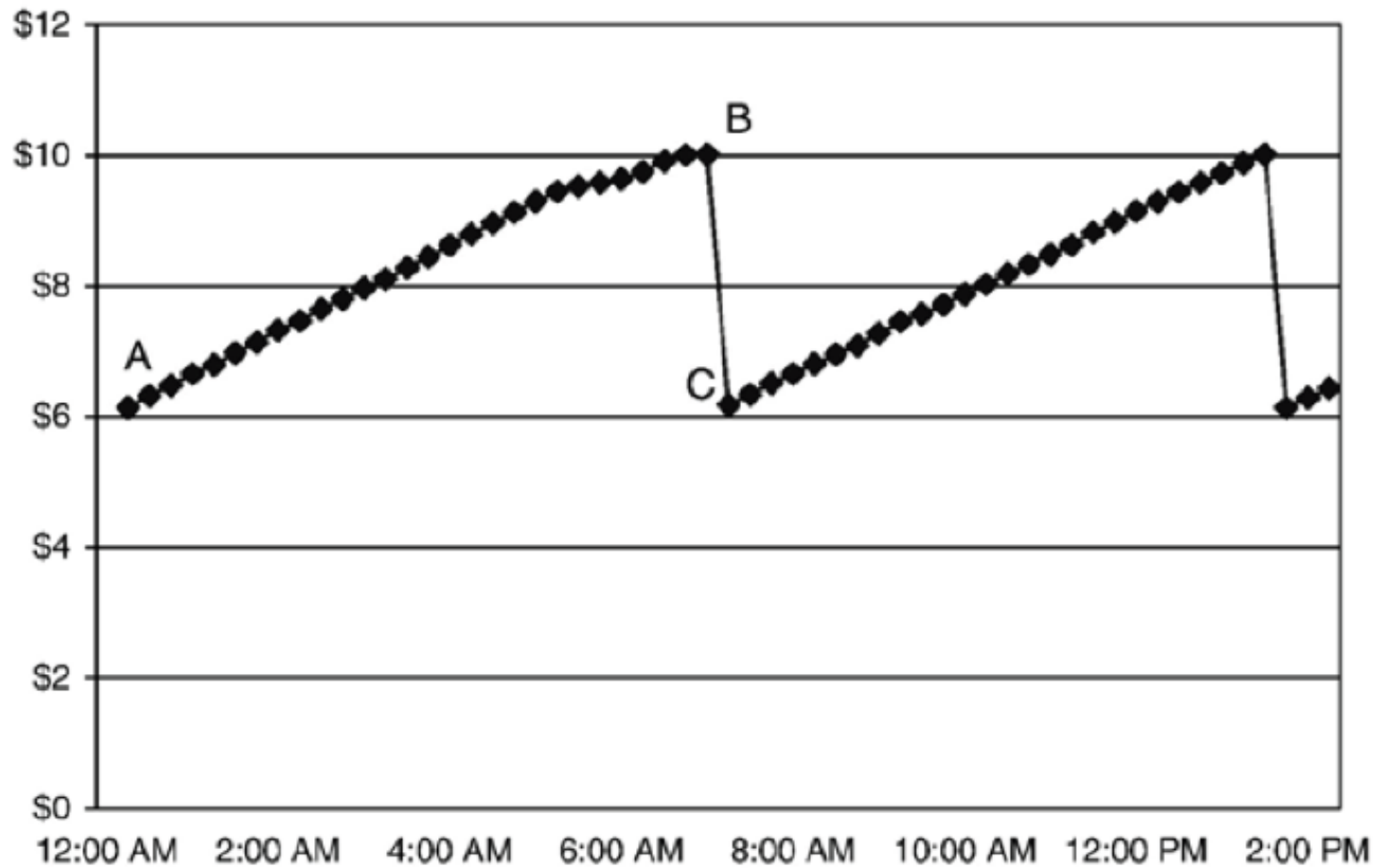
Winner Determination

- Who should go into which slot?
- Winner determination = find efficient allocation
 - Let $x_i \in M \cup \{\phi\}$ denote which slot is assigned to i
 - Maximize $\sum_{i=1}^n v_i(x_i)$
- Because of separability assumption...

Theorem 7.1: *In the separable click-through model, allocating slots in order of decreasing $q_i w_i$ is efficient.*

- \rightarrow *Winner determination in $O(n \log n)$*

Early Auctions: First-Price



Later Auctions: Second-Price

- **Generalized-Second Price Auction:**
pay an amount just high enough, such that a bid of that amount would have gotten you the same slot
- Removes incentives for continually updating bids
- Less load on auction servers

Rank-by-bid vs. Rank-by-revenue

- Rank-by-bid: allocate in order of decreasing per-click bid price

$$b_1 \geq b_2 \geq \dots \geq b_n$$

- Rank-by-revenue: allocate in order of decreasing bidder effect x per-click bid price

$$q_1 b_1 \geq q_2 b_2 \geq \dots \geq q_n b_n$$

Generalized Second-Price Auction (GSP)

Definition 7.2 (Second-Price Rank-by-revenue Auction). Assume $q_1 b_1 \geq q_2 b_2 \geq \dots \geq q_n b_n$. Allocate slots to bidders in order of decreasing bidder effect \times per-click bid price (breaking ties at random), so that slot 1 goes to bidder 1, slot 2 to bidder 2, and so forth. Upon receiving a click on an ad, charge the bidder associated with the ad a price p_i such that $q_i p_i = q_{i+1} b_{i+1}$.

- Assume: $q_i = 1$ for all bidders
- Now expected utility: $v_1 = s_1(w_1 - p_1)$
- Construct an example that shows:
GSP is not truthful!
 (i.e., truthful bidding is not a dominant strategy)

- Now what?

→ Bayes-Nash Equilibrium Analysis.

	s_i	w_i	p_i	u_i
1			↗	
2			↗	
3			↗	
4			↘	0

Equilibrium Analysis of GSP

- Assume: all bidders know each others' values!
 - Why is this justified?
 - Why do we make this assumption? [Q2]
- Thus, we can use Nash equilibrium (NE) instead of Bayes-Nash equilibrium (BNE)

Nash Equilibrium of a Slot Auction

Definition 7.3 (Nash Equilibrium of a Slot Auction). *Bid profile (b_1^*, \dots, b_n^*) is a Nash equilibrium of a slot auction if, for all i :*

$$u_i(b_i^*, b_{-i}^*) \geq u_i(b'_i, b_{-i}^*), \quad \text{for all } b'_i \neq b_i^*, \quad (7.4)$$

where the utility is defined in terms of the allocation and payment rule.

Nash Equilibrium of GSP Auction

Definition 7.4 (Nash Equilibrium of GSP Slot Auction). Assume without loss of generality $b_1^* \geq \dots \geq b_n^*$, and that ties in bid values are broken in this order for the purpose of allocating slots. In particular, advertiser 1 is allocated slot 1, advertiser 2 slot 2, and so forth. For bid profile $b^* = (b_1^*, \dots, b_n^*)$ to be a Nash equilibrium of the GSP slot auction we need:

- For $1 \leq i \leq m$ (agents allocated some slot):

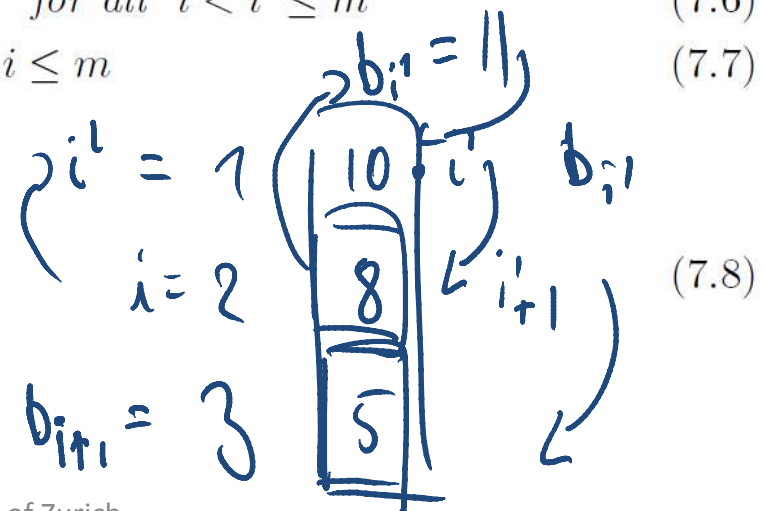
$$s_i(w_i - b_{i+1}^*) \geq s_{i'}(w_i - b_{i'}^*), \quad \text{for all } 1 \leq i' < i \quad (7.5)$$

$$s_i(w_i - b_{i+1}^*) \geq s_{i'}(w_i - b_{i'+1}^*), \quad \text{for all } i < i' \leq m \quad (7.6)$$

$$s_i(w_i - b_{i+1}^*) \geq 0, \quad \text{for all } 1 \leq i \leq m \quad (7.7)$$

- For $m < i \leq n$ (unallocated agents):

$$b_m^* \geq w_i$$

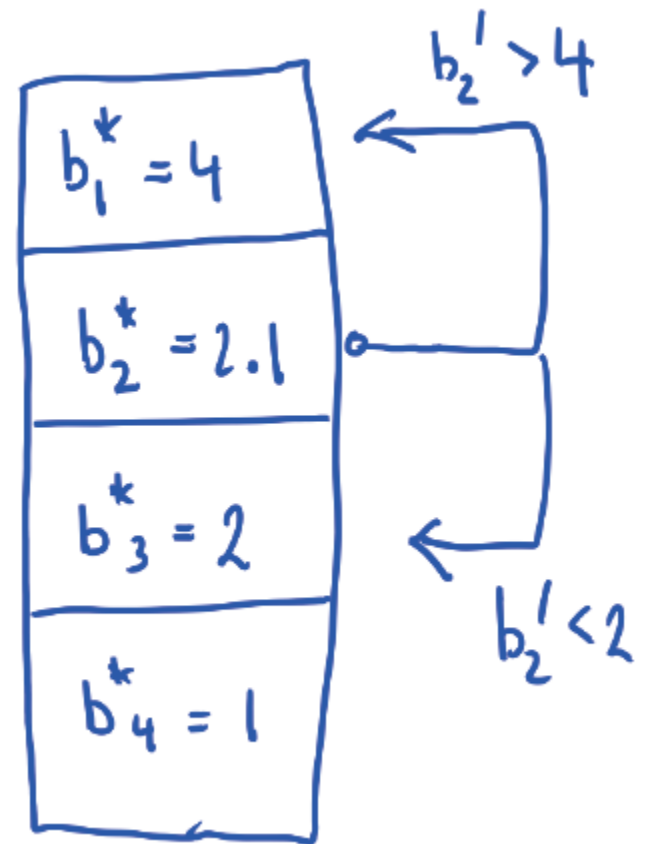


Example NE of GSP Auction

slot	w_i	b_i^*	s_i	utility
1	4	4	0.2	$0.2(4-2.1)=0.38$
2	10	2.1	0.18	$0.18(10-2)=1.44$
3	2	2	0.1	$0.1(2-1)=0.1$
-	1	1	-	0

$$b \geq 4 \rightarrow 0.2(10-4) = 1.2$$

$$\Rightarrow b \geq 1 \rightarrow 0.1(10-1) = 0.9$$



Motivating Envy-Free Equilibria

- Why study envy-free equilibria? [Q4]
 - Set of NE is complicated in GSP!
 - Every envy-free equilibrium is also a NE [Q3]
 - Envy-free equilibria are efficient!
 - If I have envy, I would increase my bid...

Envy-Free Equilibrium

Definition 7.5 (Envy-free Outcome). *The outcome of a slot auction is envy-free if, for all i :*

$$v_i(x_i) - t_i \geq v_i(x_{i'}) - t_{i'} \quad \text{for all } i' \neq i, \quad (7.9)$$

where $v_i(j)$ is bidder i 's value for slot $j \in \{1, \dots, m\} \cup \{\phi\}$.

Definition 7.6 (Envy-free equilibrium of GSP slot auction). *Assume without loss of generality $b_1^* \geq \dots \geq b_n^*$, and that ties in bid values are broken in this order for the purpose of allocating slots so that advertiser 1 gets slot 1, advertiser 2 slot 2, and so forth. For bid profile $b^* = (b_1^*, \dots, b_n^*)$ to be an envy-free equilibrium of the GSP slot auction we need:*

- For $1 \leq i \leq m$ (agents allocated some slot):

$$s_i(w_i - b_{i+1}^*) \geq s_{i'}(w_i - b_{i'+1}^*), \quad \text{for all } i' \in \{1, \dots, m\}, i' \neq i \quad (7.10)$$

$$s_i(w_i - b_{i+1}^*) \geq 0 \quad (7.11)$$

- For $m < i \leq n$ (unallocated agents):

$$b_{m+1}^* \geq w_i$$



$$(7.12)$$

Results about Envy-Free Equilibria

Theorem 7.3. *Any envy-free equilibrium of the GSP auction is also a Nash equilibrium of the GSP auction.*

- Proof in lecture notes...

Theorem 7.4. *An envy-free outcome in a slot auction is efficient (i.e., allocates slots to maximize total value.)*

- Proof in homework assignment...