Computation and Economics - Spring 2012
Assignment #2: Game Theory

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Out Monday, February 27, 2012
Due **12:15** sharp: **Thursday, March 8, 2012**
Submissions should be made in writing before the beginning of the section.
We strongly encouraged typed submissions.

[**Total: BSc 100 Points, MSc 120 Points**] This is a single-person assignment. Points will be awarded for clarity, correctness and completeness of the answers. You are free to discuss the assignment with other students. However, you should not share (even partial) answers with each other, and **copying will be penalized**.

1. [**43 Points**] Iterated elimination.

   (a) [**7 Points**] What strategies survive iterated elimination of strictly-dominated strategies in these normal-form games?

   i. [**3 Points**] Game 1:

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

   ii. [**2 Points**] Game 2:

	A	B	C
X	4,5	3,3	4,4
Y	1,2	2,7	3,5
Z	3,1	4,1	3,2

   iii. [**2 Points**] Two cavewomen are looking for food: Each decides idependently wether to hunt a Mammoth or go fishing. Fishing is easy and there are plenty of fish, hence going fishing will definitely yield food for 1 week. A Mammoth-hunt is successful only if both go hunting. In case of success, it yields 5 weeks worth of food for each hunter. Trying to hunt a Mammoth alone will fail.
   Provide the payoff matrix for this game and explain which strategies survive iterated elimination.

   (b) [**10 Points**] What are the pure strategy Nash equilibria of Game 1? Find a non-trivial mixed-strategy NE, i.e., with support of two or more actions.

   (c) [**14 Points**] Let $G = \langle N, (A_i), (u_i) \rangle$ be a Normal-form game. Show that if strategies $a^* = (a_1^*, \ldots, a_n^*)$ are a Nash equilibrium, then they survive iterated elimination of strictly dominated strategies. (*Hint: Proceed by contradiction. See Definition 2.5 of the lecture notes for a guide on the notation.*)

(d) **[7 Points]** In terms of m, the maximum number of actions, and n, the number of agents, what is the worst-case run time of iterated elimination of strictly dominated strategies? (*Hint: It is ok to assume that the comparison of two distinct action profiles* (a_i, a_{-i}) *and* $(\tilde{a}_i, a_{-i})$ *can be done in constant time.*)

(e) **[5 Points]** Consider a variation of iterated elimination that relies only on *weak* dominance; i.e., a_i is weakly dominated if there exists some $a'_i \in R_i$ such that $u_i(a_i, a_{-i}) \leq u_i(a'_i, a_{-i})$ for all $a_{-i} \in R_{-i}$ and $u_i(a_i, a_{-i}) < u_i(a'_i, a_{-i})$ for at least one $a_{-i} \in R_{-i}$. Construct an example game that shows that when using iterated elimination of weakly dominated strategies, then some Nash equilibrium may be eliminated.

2. **[14 Points]** Pareto optimality.

(a) **[9 Points]** In the lecture notes we show that a mixed-strategy equilibrium for *Matching Pennies* is each agent playing 'heads' with probability $\frac{1}{2}$.

i. **[7 Points]** Prove that this is the only mixed-strategy Nash equilibrium. (*Hint: Assume that there exists another NE with* $q := P[\text{COL plays heads}] \neq \frac{1}{2}$, *then try to maximize the expected utility of ROW.*)

ii. **[2 Points]** Explain why this NE satisfies Parteo optimality.

(b) **[5 Points]** Consider the game of *Chicken*.

i. **[2 Points]** Name the pure strategy profiles in *Chicken* that are Pareto optimal.

ii. **[3 Points]** The mixed strategy profile $\left(\left(\frac{2}{3}, \frac{1}{3}\right), \left(\frac{2}{3}, \frac{1}{3}\right)\right)$ is a NE for *Chicken*. Is it also Pareto optimal? Explain your answer.

3. **[11 Points]** Extensive-form.

Imagine a box of 100 chocolate candy, where the single items can not be split. Suppose that two players share this box as follows: First, player 1 proposes a division, then player 2 responds either "yes" or "no." If he says "yes" then the division is implemented; otherwise, no player receives anything. Each player prefers more chocolate candy to less.

(a) **[3 Points]** Formulate this as an extensive-form game using the tree notation.

(b) **[5 Points]** Find all subgame perfect equilibria of the game.

(c) **[3 Points]** Also provide a Nash equilibrium that is not subgame perfect.

4. **[22 Points, MSc +20 Points]** Folk theorems and repeated games.

(a) **[15 Points]** Show that grim trigger in Fig 3.7 (c) from Chapter 3 is a Nash equilibrium for some discount factor $\delta < 1$.

(b) **[MSc 20 Points]** Show that the *Tit-for-Tat* strategy is not a subgame-perfect Nash equilibrium of the *repeated Prisoner's Dilemma* for any discount factor $\delta < 1$. (*Hint: Proceed by contradiction: To prove this we assume that TfT is a subgame-perfect strategy, i.e. it is a best response to any subgame. We select two specific subgames and in each consider a single deviation by player 1. From the principle of one deviation and because* *of the assumptions, the deviations cannot lead to higher expected outcomes for player 1. From this we get contradicting conditions on the discount factor δ. You may need to consider histories that do not occur in equilibrium.*)

(c) **[7 Points]** Show that the grim trigger strategy from Figure 3.7 (c) is not a subgame-perfect equilibrium for any $\delta < 1$. (*Hint: Use the principle of one deviation.*)

5. **[10 Points]** P2P File Sharing

(a) **[7 Points]** The strategic-piece-revelation strategy in the BitTorrent protocol uses "under-reporting" of pieces. Consider instead a strategy based on "over-reporting" pieces, i.e., a client reporting to have pieces that it doesn't actually have. Provide some intuition for why such a strategy might make sense. Next, explain why such a strategy would be unlikely to work in practice, i.e., provide a vulnerability?

(b) **[3 Points]** The strategies in file-sharing games are provided by software, with new clients (= strategies) such as BitThief released over time. Suppose that a client is universally adopted, and even proved to be a Nash equilibrium with itself. Why might you still worry this is insufficient to provide stability of the ecosystem?