

Lecture #12: Social Choice

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Housekeeping

- Questions? Concerns?
- Homework assignments
 - Mturk assignment: Mike found the solution!
→ need to create a “sandbox requestor account”
 - VISA information for Mturk assignment:
- Course evaluation: before the break!

Recap: Human Computation

Definition 11.1 (Human Computation). *Given a computational problem from a requester, design a solution using both automated computers and human computers (Law and von Ahn, 2011).*

- **Crowdsourcing** is a type of outsourcing tasks that are traditionally performed by an employee or contractor, to an undefined, large group of people or community (a crowd) through an open call.
- Crowdsourcing markets (e.g., Mturk, Crowdfunder,...) are a great playing field to experiment with:
 - Different work allocation mechanisms
 - Different payment mechanisms
 - Different voting rules/social choice mechanisms

Duolingo

- <http://duolingo.com>

Today: Social Choice

The Set-up

- A set of alternatives $A = \{a, b, c, \dots\}$
- A set of agents $N = \{1, 2, \dots, n\}$
- Each agent i has a strict preference ordering $P_i \in L(A)$ over alternatives; for example:

$$a \succ_1 b \succ_1 c$$

$$b \succ_2 c \succ_2 a$$

$$c \succ_3 b \succ_3 a$$

- We let $P = (P_1, P_2, \dots, P_n) \in L(A)^n$ denote the preference profile (the joint preferences)
- **Social choice rule:** select best alternative $f(P) \in A$
- **Social ranking rule:** find best preference order $r(P) \in L(A)$

Mechanism Design vs. Social Choice

Theorem 12.2 (Gibbard-Satterthwaite). *If $|A| \geq 3$, any social choice rule that is onto A and strategyproof is dictatorial.*

- We have seen:
 - With $|A|=2$, majority rule is strategyproof.
 - With $|A| \geq 3$, Plurality can be manipulated
 - With $|A| \geq 3$, Borda can be manipulated
 - In fact: all interesting voting rules can be manipulated
- But: we still have to make a decision, even when $|A| \geq 3$. → Social Choice Theory

The two Different Perspectives

- The subjective viewpoint:
 - With perfect information → still disagreement
 - Difference in votes due to different preferences
- The objective viewpoint:
 - With perfect information → full agreement
 - Difference in votes due to noise
- Examples...

Properties of Voting Rules

- Different voting rules still have different properties!
- Which ones do we want?
- We liked the majority rule for $|A|=2$, because...?
...the majority of voters will agree that the winner is indeed the best!
- Generalize to $|A| \geq 3$...the alternative that beats every alternative by simple majority

Condorcet Consistency

$$a \succ_1 b \succ_1 c$$

- Condorcet Paradox:

$$b \succ_2 c \succ_2 a$$

$$c \succ'_3 a \succ'_3 b,$$

- **Condorcet Consistency:** a social choice rule is *Condorcet consistent* if it selects as the best alternative an alternative that beats (or ties) all others in pairwise comparisons, *when one exists*.

Some Popular Voting Rules

Definition 12.3 (Plurality). *Given $m \geq 3$ alternatives, each agent votes for its most preferred alternative. The alternative with the most votes is selected, breaking ties at random.*

Definition 12.4 (Plurality-with-run-off). *Given $m \geq 3$ alternatives, each agent votes for its most preferred alternative. Choose the two alternatives with the most votes (breaking ties at random), and run a second round of majority voting on these top two.*

Definition 12.5 (Plurality-with-elimination). *Given $m \geq 3$ alternatives, each agent votes for its most preferred alternative. Eliminate the alternative with the fewest votes (breaking ties at random). Each agent who cast a vote for the eliminated alternative casts a new vote. Repeat until only one alternative remains.*

Definition 12.6 (Borda). *Each agent submits a preference order on the m alternatives. Based on this, a score of $m - 1$ is assigned to the top alternative, $m - 2$ to the second, \dots , down to 0 points to the last alternative. The alternative selected is the one with maximum total score, summing up across all agents (and breaking ties at random).*

Positional Scoring Rules

Definition 12.7 (positional scoring rule). *A positional scoring rule gives a score of s_1, \dots, s_m to the alternative in position $1, \dots, m$ of an agent's preference order, with $s_j \geq 0$ and $s_1 \geq s_2 \geq \dots \geq s_m$ and $s_1 > s_m$. The selected alternative by the rule is the one with the maximum total score.*

- **Theorem:** No positional scoring rule is Condorcet consistent on three or more alternatives.
- But:
 - Positional scoring rules are simple
 - Widely used in practice

Axiomatic Approach

Definition 12.8 (unanimity). *A social ranking rule r satisfies unanimity if, given preference profile P , whenever all agents i agree that $a \succ_i b$, then $a \succ_r b$ in the social preference order $P_r = r(P)$.*

Definition 12.9 (independence of irrelevant alternatives (IIA)). *A social ranking rule r is independent of irrelevant alternatives if for any $a, b \in A$ and any two preference profiles P, P' that leave the preference between a and b unchanged for all agents (but can change other preferences), then the social preference order between a and b is unchanged.*

Theorem 12.3 (Arrow's impossibility theorem). *If $|A| \geq 3$, any social ranking rule r that satisfies unanimity and IIA is dictatorial.*

- Says nothing about strategy-proofness!!
→ Need to relax IIA → local consistency

Relaxing IIA

- Interval: some set of alternatives $A' \subset A$ form an interval given preference profile P and social ranking rule r , if alternatives A' are adjacent in the social rank order $r(P)$.

- Example: Apply the Borda rule

$$3 \quad @ \quad a \succ b \succ c \succ d$$

$$2 \quad @ \quad b \succ c \succ d \succ a$$

$$2 \quad @ \quad c \succ d \succ a \succ b$$

- The social ranking is: $c \succ_r b \succ_r a \succ_r d$
- Now, drop alternative d :

Local Consistency

Definition 12.10 (local consistency). *A social ranking rule r satisfies local consistency if, for any preference profile P leading to social preference order \succ_r , applying r to a domain restricted to any set of alternatives A' on an interval in \succ_r leaves the social preference order on A' unchanged.*

- Weaker than IIA
- Borda fails even local consistency
- Are there voting rules that are Condorcet consistent and satisfy local consistency?

→ Idea: select as the winning alternative one that is as close as possible to being a Condorcet winner!

Swap Distance

- The swap distance between P_i and P_i' is the minimum number of swaps of the position of adjacent alternatives to obtain P_i' from P_i .
- Example:

Dogson Rule (1/2)

Definition 12.11 (Dodgson rule). *Each agent submits a preference order on the m alternatives, providing preference profile P . Select as the winner an alternative that can be made a Condorcet winner in a preference profile P' that minimizes the swap distance from P .*

- **Example:**
 $a \succ_1 e \succ_1 d \succ_1 c \succ_1 b$
 $e \succ_2 d \succ_2 c \succ_2 b \succ_2 a$
 $b \succ_3 a \succ_3 c \succ_3 e \succ_3 d$
 $b \succ_4 e \succ_4 d \succ_4 a \succ_4 c$
 $c \succ_5 d \succ_5 a \succ_5 b \succ_5 e$

- Alternatives a, b or e all have a Dogson score of 2
- To make a Condorcet winner:
→ Change agents 2 and 4

Dogson Rule (2/2)

- **Theorem:** The Dogson rule is Condorcet consistent!
- **Proof:** ?

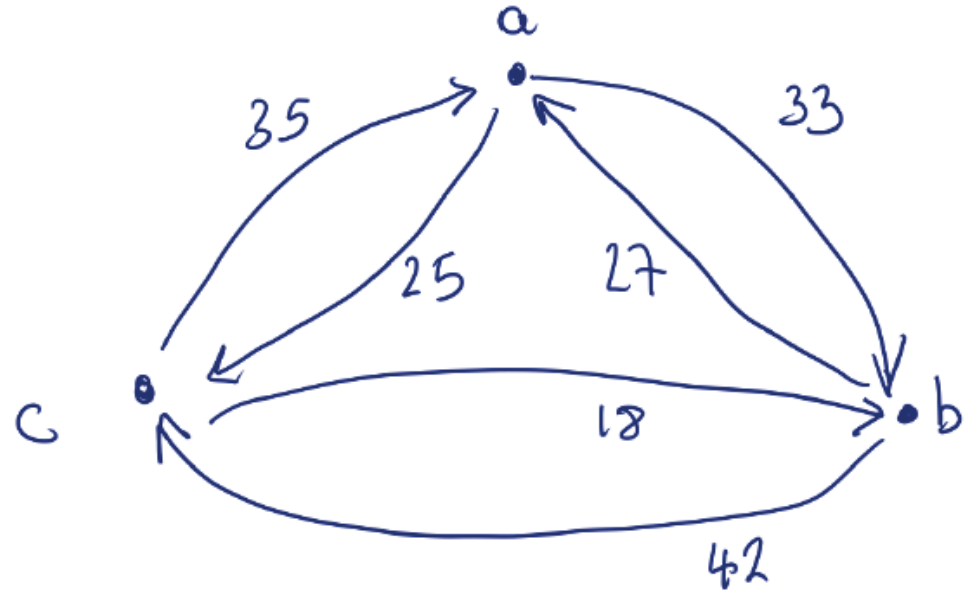
The Kemeny Rule

Definition 12.12 (Kemeny rule). *Each agent submits a preference order on the m alternatives, providing a preference profile P . Select as the social preference order P_r a preference order such that a profile that consists of P_r for every agent minimizes the swap distance from P . Select as the best alternative the best alternative in P_r .*

- Swap distance (P, P') = number of pairwise disagreements (P, P')
 - Number of disagreements on (x,y) = number of agents – number of agreements on (x,y)
- Kemeny: maximize total number of pairwise agreements

Kemeny Rule: Example

23	@	$a \succ b \succ c$
17	@	$b \succ c \succ a$
2	@	$b \succ a \succ c$
10	@	$c \succ a \succ b$
8	@	$c \succ b \succ a$



- No Condorcet winner: a beats b , b beat c , c beats a
- Compute total pairwise support >

$a \succ b \succ c$	$a \succ c \succ b$	$b \succ a \succ c$	$b \succ c \succ a$	$c \succ a \succ b$	$c \succ b \succ a$
100	76	94	104	86	80

Kemeny Rule (ct.)

- **Theorem:** The Kemeny rule is Condorcet consistent!
- Proof:

More Properties of the Kemeny Rule

- **Neutrality:** the outcome is invariant to the identity of the alternatives.
- **Anonymity:** the outcome is invariant to the identity of the agents.

Definition 12.13 (reinforcement). *A social ranking rule r satisfies reinforcement if when two pools of agents with preference profile P and P' respectively generate the same social preference order $P_r = r(P) = r(P')$ then then social preference order is the same after the pools are merged, with $P_r = r(P \cup P')$.*

- **Theorem:** The Kemeny rule is the unique social ranking rule that satisfies anonymity, neutrality, unanimity, reinforcement and local-consistency.

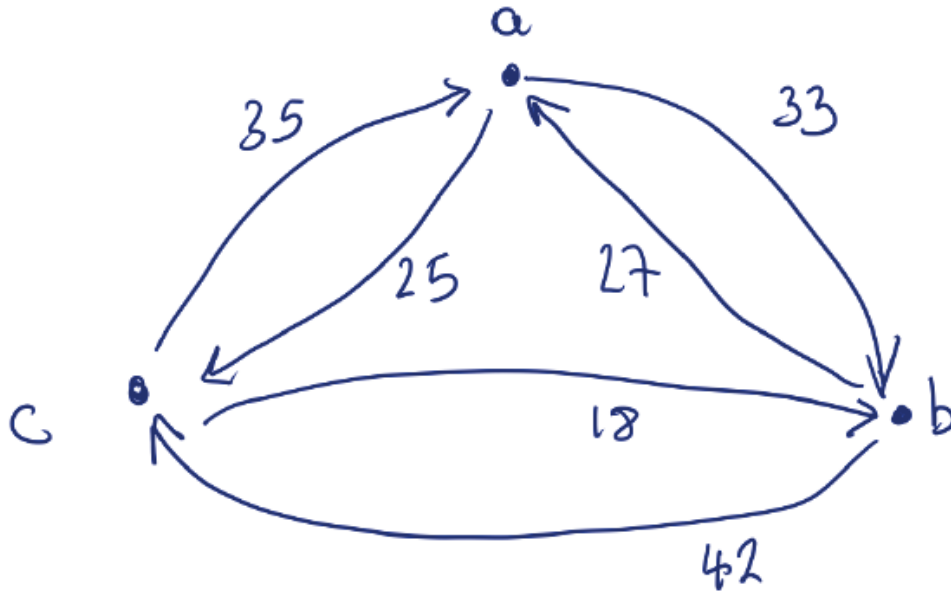
The Statistical View

- Consider the “objective” viewpoint:
Any difference in votes is due to noise
 - The Kemeny rule identifies the alternative that is “most likely” to be the best alternative!
- Strong statistical support. This is good, in particular when there is no strategic voting!!

Computational Considerations

- Winner determination = which alternative (or which ranking) gets selected?
- Many voting rules are easy to compute
 - Majority
 - Plurality
 - Borda
- But Kemeny is more difficult...

Computing Kemeny



- For every pair of alternatives, select one edge, such that the path is acyclic
- Maximize the total weight of the selected edges

$$\max_{x_{jk}: j \neq k} \sum_{j, k: j \neq k} w_{jk} x_{jk}$$

$$\text{s.t. } x_{jk} + x_{kj} = 1, \quad \text{for all } j, k, j \neq k$$

$$x_{jk} + x_{kl} + x_{lj} \leq 2, \quad \text{for all } j, k, l, \text{ and } j \neq k \neq l$$

$$x_{jk} \in \{0, 1\}, \quad \text{for all } j, k, j \neq k$$

Complexity of Kemeny Rule

- Can be formalized as IP \rightarrow can compute in worst-case exponential time
- It is known that computing Kemeny rankings is NP-hard \rightarrow cannot do better than worst-case exponential time
- In practice: 25 alternatives in 5 seconds
- Some very hard instances with 40 alternatives

The Schulze Rule

- Alternative to Kemeny
- Can be computed in polynomial time $O(m^3)$.
- Also Condorcet consistent.

Course Evaluation



University of
Zurich^{UZH}

Academic Program Development

Your course – your opinion

Online questionnaire for Bachelor and Master's
students in this course

from 23 April – 6 May 2012

You will receive an e-mail with further information about the survey when it begins on 23 April 2012.

Thank you for participating!

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