Exercise 3: Logic and ANTLR

Formal Methods II, Fall Semester 2013

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Send your solutions to: tobias.klauser@uzh.ch or deliver them in the class.

Logic

Propositional logic

1. (2 points) Show that:
   (a) $p \rightarrow (q \rightarrow p)$ is a tautology.
   (b) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ is a tautology.
   (c) $(\neg q \rightarrow \neg p) \leftrightarrow \neg p \wedge \neg(p \vee q)$ is not a tautology.
   (d) $\neg(p \vee \neg p)$ is always false (contradiction).
2. (3 points) Express the following logical statements with $\wedge$, $\vee$ and $\neg$ (and obviously a, b and c):

   (a) $c \rightarrow (b \rightarrow a)$ (b) $(b \rightarrow a) \leftrightarrow c$ (c) $a \veebar b$

   Note : $a \veebar b$ is true if either a or b but not both are true (exclusive disjunction).
3. (1 point) Is it possible to express all statements of propositional calculus using only $\vee$ (or) and $\neg$ (not)? If yes, demonstrate how; if no, give a counter example.
4. (2 points) Transform the following formula into conjunctive normal form by using syntactic operations:

   $$(C \vee D) \rightarrow (C \wedge D)$$

   You mustn't use a truth table in this exercise. However, you may simplify your formula by leaving out tautologies.

Predicate logic

5. (4 points) Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define and state in your answer):

   (a) The highest building in Manhattan is higher than the highest building in Zurich.

   (b) There is a young person who owns a fast and safe car.

   (c) There is a person who likes all mopeds which are not slow.

   (d) No person likes a slow car.

Bonus Chuck Norris saves only those people who don't save themselves.

6. (2 points) Let us consider the following formula which is valid in number theory (arithmetics):

$$(\forall x)(x \neq 0 \rightarrow (\exists y)(x = S(y)))$$

   where $(x \neq 0)$ is a shorthand for $\neg(x = 0)$ and $S(y)$ is the successor function which returns a natural number equal to $y + 1$.

   (a) Look at the definition of a term and a formula in the script and show how the formula was formed from its atomic formulas.

   (b) Can you state in natural language what the formula says?

7. (3 points) Show that (using the rules from the script) the following expressions with predicates P and Q

   (a) $\neg\exists x(\forall y(P(x, y)))$ and $\forall x(\exists y(\neg P(x, y)))$ are equivalent.

   (b) $\exists x(P(x)) \wedge \exists x(Q(x))$ and $\exists x(P(x) \wedge Q(x))$ are not equivalent (give a counter example).

   (c) $\forall x(P(x)) \wedge \exists x(Q(x))$ and $\forall x(\exists y(P(x) \wedge Q(y))$ are equivalent.

ANTLR Flashback

8. (3 points) Formal languages with ANTLR. Download ANTLRWorks *version 3* from `http://antlr3.org/works/` to solve this exercise.

   The grammar defined in the file `boolean.g` (contained in the exercise material) shall accept valid expressions of boolean algebra. The string `&&` is used for conjunctions and `||` for disjunctions. Furthermore, `!` represents a negation. `T` and `F` are used for true and false, respectively.

   (a) There is something wrong with the conjunction that prevents the interpretation of strings like `T&&F`. Sort out this issue.

   (b) Implement the implication operator using the string `->` and make sure that it has a lower precedence than the disjunction operator while doing so. Also add the equivalence operator `<->` with an even lower precedence in an equal manner.

   (c) Modify the grammar such that it accepts any lower case alphabetic string to allow for variables in your expressions.